EXAMPLES

OF

ASTRONOMIC AND GEODETIC

CALCULATIONS

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CALCULATIONS

FOR THE USE OF

LAND SURVEYORS

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GOVERNMENT OF CANADA

DEPARTMENT OF THE INTERIOR

DOMINION LANDS OFFICE

BOARD OF EXAMINERS FOR DOMINION LAND SURVEYORS

Moved by Mr. Andrew Russell, seconded by Major Webb and resolved:

" That the Secretary be directed to communicate to " Capt. DEVILLE the appreciation of the value of his " work by the board, as an aide memoire to Surveyors " in the field."

" The Board is particularly pleased with the solution " given by him, for practical purposes, of certain Geo- " detic problems by the method set forth, depending on " the convergence of Meridians; and also of Capt. DE- " VILLE's labor in calculating the original and very " useful tables appended to the work."

ABBREVIATIONS.

M. T.—Mean time.

A. T.—Apparent time.

S. T.—Sidereal time.

Gr.—Greenwich.

H. C. R.—Horizontal circle reading.

ch, chs,—Gunter's chain or chains.

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PREFACE.

In almost all countries, the duties of a Land Surveyor consist only of laying out and dividing land; geodetic surveying belongs to the business of the civil and military engineer corps, composed of men who have mastered the highest branches of science. We have no such bodies here, and the Land Surveyor is often called upon to undertake extensive topographic surveys and those of geographic exploration. But, in what books is he to find the knowledge necessary to fulfil those duties? I was often asked the question and could not answer; the works on the subject, intended for eminent mathematicians, are too abstruse; what is needed, and which I have endeavoured to prepare, is a treatise presenting under a practical and elementary form, the solution of the problems most frequently met with in practice. I had to alter some of the usual methods, and in some instances to devise entirely new ones, in order to maintain the same simplicity through out. Having only my own experience to direct me in this work, I know it presents many imperfections, I hope nevertheless, that as a whole, it will prove useful and acceptable to surveyors generally.

Great extension is given to the chapter on Azimuth. By the methods shewn, and with the aid of the tables annexed, it will be possible to find the direction of the meridian at almost any time of the day or night.

Instead of the long and tedious formulae of geodesy, I have given some simple and expeditious ones, depending on the convergence of meridians. The results are nearly the same as with the series deduced from the precise formulae, when such terms are neglected as contain powers of the distance above the second. This is sufficiently accurate for all practical purposes.

At places without telegraphic connection, surveyors are sometimes requested to establish sun dials. Chapter VI explains the most convenient processes to construct them.

The tables have been carefully computed with the latest numerical values; they have been, like the work generally, specially devised for Canada.

This work must be used only in the Northern hemisphere, as many of the methods would be wrong South of the Equator.

CHAPTER I.

DEFINITION AND PRELIMINARY OPERATIONS—EPHEMERIS. (1)

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1. Division of the circle.

The circle is divided into 360 degrees or 24 hours. The degree or hour is divided into 60 minutes, the minute into seconds and the second into 60 thirds. This last division is seldom used, the decimals of the second being taken instead.

An angle is composed of a number of degrees and fractions of a degree or hours and fractions of an hour. Thus we say: an angle of 60 degrees 30 minutes and 45 seconds or an angle of 4 hours 2 minutes and 3 seconds, and they are written thus:

$$60^\circ\ 30'\ 45''$$
$$4^h\ 02^m\ 03^s$$

(1) The reader will find, at the end of Nautical Almanac some explanations on the elements of astronomy and the use of the Ephemeris and tables.

We have, according to definition:

$$
\begin{aligned}
360^\circ &= 24^h \\
90^\circ &= 6^h \\
15^\circ &= 1^h \\
1^\circ &= 4^m \\
15' &= 1^m \\
1' &= 4^s \\
15'' &= 1^s
\end{aligned}
$$

It will be observed that the minutes and seconds of arc have not the same value as the same parts of the hour, although having the same denomination.

2. To convert an arc into time.

Multiply by 4, the product of the degrees in the arc will give minutes of time, that of the minutes, seconds, and that of the seconds, thirds. Divide, then, the number of thirds by 60, to reduce them to decimals of a second.

Example 1.—To convert into time 5° 12′ 06″

	$5^\circ \times 4$	gives	20^m		
	$12' \times 4$	"		48^s	
	$6'' \times 4$	"			24^t
	$5^\circ\ 12'\ 06''$	=	20^m	48^s	24^t
or		=	20^m	$48^s, 4$	

Example 2.—To convert into time 76° 43′ 28″

	$76^\circ \times 4$	gives	304^m		
	$43' \times 4$	"		172^s	
	$28'' \times 4$	"			112^t
	$76^\circ\ 43\ 28''$	=	304^m	172^s	112^t
or		=	$5^h\ 06^m$	53^s	87

3. To convert time into arc.

Reduce the hours to minutes, add the given number of minutes and divide by 4, the result will be degrees. Reduce the remainder to seconds, add the given number of seconds and divide by 4, the quotient will give minutes, and generally a decimal fraction of a minute, which may be reduced to seconds by multiplying by 60.

Most of the tables of logarithms contain tables for converting time into arc and conversely, they require no explanation.

Example 1.—To convert into arc $10^h\ 29^m\ 58^s$

	$629^m : 4$	gives	$157°$	remainder 1^m
	$118^s : 4$	"	$29'5$	
	$10^h\ 29^m\ 58^s$	$=$	$157°\ 29'.5$	
or		$=$	$157°\ 29'\ 30''$	

Example 2.—To convert into arc $7^h\ 51^m\ 27^s$

	$471^m : 4$	gives	$117°$	remainder 3^m
	$207^s : 4$	"	$51'.75$	
	$7^h\ 51^m\ 27^s$	$=$	$117°\ 51'.75$	
or		$=$	$117°\ 51'\ 45''$	

4. Time.

The transit, meridian passage or culmination of a heavenly body is the instant when that body is on the

meridian. Transits are divided into upper and lower, according as they take place above or below the pole.

A day is the interval of time between two successive upper transits of a heavenly body. It is a solar, lunar or sidereal day, according as the body is the sun, the moon or a star. (1)

Sidereal days are nearly equal, but true solar days are not, as the motion of the sun is not uniform. To obtain a uniform measure of time, astronomers have devised a mean sun, moving uniformly. The interval between two transits is a mean solar day for the mean sun, and an apparent or true solar day for the true sun.

Time is known as mean, apparent, or sidereal, according as it is reckoned in mean, apparent, or sidereal days.

The astronomical day begins at noon and is divided into 24 hours, numbered from 1 to 24.

The civil day begins at midnight, twelve hours before the astronomical. It is divided into two periods of 12 hours each, marked A.M. from midnight to noon and P.M. from noon to midnight. 0^h, astronomical time, corresponds to noon the same day of the month and 12^h, astronomical time, to midnight at the beginning of the following day. Civil time is, usually, mean solar time.

5. To convert astronomical into civil time.

When the number of hours is less than 12, add the designation P. M.

(1) More accurately speaking, a sidereal day is the interval between two upper transits of the vernal equinox.

When it is more than 12, subtract 12^h, make its the designation A.M. and add one to the day of the month.

Example 1.—What is the civil time on Sept. 9 at 18^h. 26^m. 23^s, astronomical time ?

Ans.—Sept. 10, at 6^h. 26^m. 23^s A.M.

Example 2.—What is the civil time on July 4 at 7^h. 45^m. 30^s, astronomical time ?

Ans.—July 4, at 7^h. 45^m. 30^s. P.M.

6. To convert civil into astronomical time.

For P.M. times, make no change, but for A.M. ones, add 12^h and subtract one from the day of the month.

Example 1.—What is the astronomical time on Aug. 4, at 9^h 35^m 43^s A.M. ?

Ans.—Aug. 3, at 21^h 35^m 43^s. astronomical time.

Example 2.—What is the astronomical time on Jan. 24, at 2^h 07^m 31^s. P.M.

Ans.—Jan. 24, at 2^h 07^m 31^s. astronomical time.

7. To find the Greenwich time corresponding to the time of another place.

When the place is West of Greenwich, add the longitude to the given time, or subtract it when it is East. If the subtraction cannot be made add 24^h or if the result is more than 24^h subtract 24^h, subtracting or adding, at

the same time, one to the day of the month. When the longitude is given in arc, it is converted into time as in Art. 2.

Example 1.—What is the Greenwich time on Oct 24, at 9^h 28^m 42^s M. T. of Quebec?

Long. of Quebec 71° 12′ 51″ W.

$$
\begin{array}{rl}
71^\circ\ 12'\ 52''\ \text{W.} = & 4^h\ 44^m\ 51^s\ \text{W. (Art. 2)} \\
\text{Quebec M. T.} = \text{Oct. 24, at} & 9\ \ \ 28\ \ \ 42 \\
\hline
\text{Greenwich M. T.} = \text{Oct. 24, at} & 14^h\ 13^m\ 33^s
\end{array}
$$

Example 2.—What is the Greenwich time on Sept. 29, at 21^h 13^m 41^s M. T. of Toronto.

Longitude of Toronto 5^h 17^m 33^s W.

$$
\begin{array}{rl}
\text{Long. of Toronto} = & 5^h\ 17^m\ 33^s\ \text{W.} \\
\text{Toronto M. T.} = \text{Sept. 29 at} & 21\ \ \ 13\ \ \ 41 \\
\hline
\text{Greenwich M. T.} = \text{Sept. 29 at} & 26\ \ \ 31\ \ \ 14 \\
\text{or} \qquad = \text{Sept. 30 at} & 2^h\ 31^m\ 14^s
\end{array}
$$

8. Given the time of a place, to find the time of another place.

Take the sum of the longitudes, if they are of different denominations, or their difference if of the same. Add it to the given time, if the second place is East of the first, or subtract if West. When the substraction is not possible or the result is more than 24^h, 24^h must be added to or taken from it, subtracting or adding, at the same time, one to the day of the month.

Example 1.—What is the Ottawa time on Oct. 27 at 3^h 27^m 30^s M. T. of Moisy P. Q.

Long. of Ottawa $5^h\ 02^m\ 54^s$ W. Long. of Moisy $4^h\ 24^m\ 12^s$ W.

Long. of Ottawa	$5^h\ 02^m\ 54^s$. W.
Long. of Moisy	4 24 12 W.
Difference	0 38 42
Moisy M. T., Oct. 27 at	3 27 30
Ottawa M. T., Oct. 27 at	$2^h\ 48^m\ 48^s$

Example 2—What is the time at Quebec on Aug. 21 at $23^h\ 47^m\ 25^s$ M. T. of Winnipeg.

Long. of Quebec $4^h\ 44^m\ 51^s$ W. Long of Winnipeg $6^h\ 28^m\ 30^s$ W.

Long. of Winnipeg	$6^h\ 28^m\ 30^s$. W.
Long. of Quebec	4 44 51. W.
Difference	1 43 39
Winnipeg M. T. Aug. 21.	23 47 25
Quebec M. T. Aug. 21.	25 31 04
or Aug. 22.	$1^h\ 31^m\ 04^s$

Example 3.—What is the time at Sydney (Australia) on June 6 at $15^h\ 23^m\ 30^s$ Montreal M. T.

Long. of Sydney $10^h\ 05^m\ 00^s$ E. Long. of Montreal $4^h\ 54^m\ 13^s$ W.

Long. of Sydney	$10^h\ 05^m\ 00^s$. E.
Long. of Montreal	4 54 13. W.
Sum	14 59 13
Montreal M. T. June 6	15 23 30
Sydney M. T. June 6	30 22 43
or June 7	$6^h\ 22^m\ 43^s$

9. Interpolation.

The quantities in the Nautical Almanac are given for certain Greenwich times. As they are continually varying, corrections are necessary to ascertain their values at any intermediate instant, and these are to be proportional to the intervals elapsed since the Nautical Almanac times. Consequently, we must first obtain the Greenwich time at the instant for which the value is required, and then, find the correction for the difference between it and the Nautical Almanac time. Supposing this quantity to be given for every Greenwich mean noon, the variation in 24 hours will be known and a simple proportion will give the correction for the elapsed time since Greenwich mean noon. It is to be added or substracted according as the quantity given in the Nautical Almanac is increasing or decreasing.

When the variation in 1 hour is given, the correction will be found by multiplying it by the number of hours and decimal of an hour elapsed since Greenwich mean noon.

Otherwise the best method is to proceed by aliquot parts and say:

> if for 24^h the variation is so much: for so many hours it will be so much; and so on.

When any of the aliquot parts are not to be used, they must be crossed out before adding.

Example 1.—To find the Sun's declination on Nov 21, 1877 at 7^h 24^m M. T. Greenwich.

We find in the Nautical Almanac, page II of the month (Right hand side): (1)

Sun's declination on Nov. 21 at 0^h M. T. Gr.			20° 01′ 16″. 3 S		
"	"	22	"	20 14 11 . 6	"

		Var. in 24^h	12′ 55″
Declination on Nov. 21, at 0^h M. T. Gr.	20°-01′-16″. S.	6	3 14
Prop. part for 7^h 24^m	3 59	1	32
Declination on Nov. 21, at 7^h 24^m M. T. Gr.	20° 05′ 15″. S.	20^m	11
		4	2
		7^h 24^m	3′ 59″

Example 2.—To find the equation of time on Dec. 14, 1877 at 0^h A. T. Sherbroke, P. Q.

Sherbrooke's Longitude, 4^h 48^m. W.

The Nautical Almanac gives for apparent noon, page I of the month (left hand side):

Equation of time on Dec. 14		4^m 58^s. 89	
			Var. in 1^h — 1^s. 2
"	" 15	4 29 89	

Sherbrooke A. T.	Dec. 14	0^h 00^m 00^s
Longitude		4 48 00 W.
Gr. A. T. at 0^h A. T. Sherbrooke Dec. 14.		4 48 00
	or	4^h, 8

(1) Page 1 of the month gives the elements for apparent noon and page II, for mean noon. When apparent time is known, page 1 is used and when mean time, page II.

Equation of time on Dec. 14 at 0^h A. T. Gr.	4^m 58^s 89	Var. in 1^h 1^s. 2
Var. in 4^h 48^m	5 76	Multiplied by 4 . 8
		9 6
		48
Equation of time on Dec. 14 at 0^h A. T. Sherbrooke	4^m 53^s 13	Var. in 4^h 48^m = 5^s 76

Example 3.—To find the sun's declination on Sept. 3, 1877, at 5^h 24^m M. T. of Montreal.

Longitude of Montreal 4^h 54^m W.

We find in the N. A. for Greenwich mean noon:

Sun's declination
on Sept. 3.—7° 25′ 49″. 7 N

Var. in 1^h = 55″ 23 (1)

" 4.—7 03 40 . 3 N

Montreal M. T.	Sept. 3,	5^h 24^m
Longitude		4 54 W.
Greenwich M. T.	Sept. 3,	10^h 18^m
	or "	10^h, 3

Sun's declination at 0^h M. T. Gr.	7° 25′ 49″. 7 N.	Var. in 1^h	55″. 23
Correction for 10^h 18^m	9 28 .9	Multiplied by	10 . 3
			165 69
			5523
Declination at 6^h 24^m Montreal M. T.	7° 16′ 20″ 8 N.	V. in 10^h 18^m	568″ 869
		or	9′ 28″. 9

(1) This variation is taken from page I of the month. It may be assumed to be the same for mean and apparent time.

10. Conversion of the simultaneous times of a place.

Different times correspond, at the same place, to the same instant. We shall consider only three of them, the mean, apparent, and sidereal time. It is often required to convert one into another. We shall give examples of different cases in succession.

11. To convert apparent into mean time and conversely.

Find the Greenwich time and the corresponding equation of time, which is to be added to or subtracted from the given time, according to the precept at the head of the Nautical Almanac's column.

When great accuracy is not required, the calculation of the Greenwich time may be dispensed with, and the equation of time taken for Greenwich noon.

Example 1.—Required the apparent time on July 20 at $3^h\ 32^m\ 28^s$ M. T. of Sherbrooke.

Long. of Sherbrooke $4^h\ 48^m$, W.

Sherbrooke M. T.	July 20,	$3^h\ 32^m$
Longitude		4 48 W
Greenwich M. T.	July 20,	$8^h\ 20^m$
	or "	$8^h\ \ 3$

Equation of time on July 20 at 0^h M. T. Gr. (To be subtracted from M. T.)	6^m 04^s 64
Var. for 8^h 20^m	1 16
Equation of time on July 20 at 3^h 32^m M. T. Sherbrooke	6 05 80
Sherbr. M. T. July 20,	3 32 28 00
Sherbr. A. T. July 20,	3^h 26^m 22^s 20

Var. in 1^h	0^s 14
Multiplied by	8 , 3
	4 2
	112
V. in 8^h 20^m	1^s 16 2

Example 2.—To find the mean time of the sun's meridian passage on Nov. 2, 1877 at Quebec. (1)

Quebec A. T.	Nov. 2,	0^h 00^m 00^s
or	Nov. 1,	24 00 00
Equat. of time to be subtracted from A.T. (2)		16 20
Mean time of the sun's meridian passage		23^h 43^m 40^s

12. To convert a sidereal interval into a mean one and conversely.

This is done by means of the tables at the end of the Nautical Almanac "For converting intervals of mean solar time into equivalent intervals of sidereal time" and "For converting intervals of sidereal time into equiva-

(1) It must be remembered that the time of the sun's transit is apparent noon.

(2) The equation of time is here taken for Greenwich apparent noon. If more accuracy be needed, it should be corrected for elapsed time since Greenwich apparent noon.

lent intervals of mean solar time" They require no explanation. (1)

13. To convert mean into the corresponding sidereal time.

Take out of the Nautical Almanac the sidereal time at mean noon, then add the proper correction for the time elapsed since Greenwich mean noon, which is equal to $9^s\ 8565$ for every hour of longitude or $3^m\ 56^s\ 6$ for 24 hours. Add the given mean time converted into a sidereal interval by means of the tables, the result is the required sidereal time. When greater than 24^h, subtract this quantity without altering the date, which is the same as for mean time, sidereal days having no particular date.

This operation and the next one might also be performed with the time of transit of the first point of Aries, given in the Nautical Almanac.

Example.—Find the sidereal time on June 11, 1877 at $17^h\ 25^m\ 00^s$ M. T. of Chicoutimi P. Q.

Long. of Chicoutimi $4^h\ 44^m\ 20^s$. W.

By table "For converting mean into sidereal intervals"

17^h	M. T.=	$17^h\ 02^m\ 47^s\ 56.$ S. T.
25^m	=	25 04 11.
$17^h\ 25^m$	M. T.=	$17^h\ 27^m\ 51^s\ 67.$ S. T.

(1) Dr. Peters Astronomiche tafeln und formeln, Wilhelm Mauke, Hamburg, contain, among many useful tables, some more convenient than those of the Nautical Almanac for those conversions.

S. T. on June 11 at 0^h M. T. Gr.	$5^h\ 19^m\ 03^s\ 22$	Var. in 24^h	$3^m\ 56^s\ 6$
Var. in $4^h\ 42^m\ 20^s$	46 71		
S. T. on June 11 at 0^h M. T. Chicoutimi	5 19 49 93	4	39 43
M. T. interval converted		40^m	6 57
into S. T.	17 27 51 67	4	0 66
S. T. on June 11 at 17^h		20^s	0 05
25^m M. T. Chicoutimi	$22^h\ 47^m\ 41^s\ 60$		
		Var. in $4^h\ 42^m\ 20^s$	$46^s\ 71$

14. To convert sidereal into the corresponding mean time.

Find as above, the sidereal time at mean noon of the place; subtract it from the given time, and reduce the remainder to a mean time interval by the Nautical Almanac table. When the subtraction is not possible, 24 hours are added to the given sidereal time.

Example.—Required the mean time on Nov. 6. 1877 at $9^h\ 13^m\ 53^s$ S. T. of Rouse's Point.

Longitude $4^h\ 53^m\ 28^s$ W.

S. T. on Nov. 6 at 0^h M. T. Gr.	$15^h\ 03^m\ 13^s\ 48$	Var. in 24^h	$3^m\ 56^s\ 6$
Correction for $4^h\ 53^m\ 28^s$	48 21		
		4	39 43
S. T. on Nov. 6 at 0^h M.		40^m	6 57
T. Rouse's Point	15 04 01 69	10	1 64
Rouse's Point S. T. (1)	33 13 53 00	3 20^s	55
		8	02
S. T. from mean noon	$18^h\ 09^m\ 51^s\ 31$		
		Var. in $4^h\ 53^m\ 28^s$	$48^s\ 21$

(1) 24^h are added to make the subtraction possible.

By table "For converting sidereal into mean time intervals."

18^h S. T. =	$17^h\ 57^m\ 03^s.07$	M. T.
9^m	8 58.53	
51^s	50.86	
0.32	0.32	
Rouse's Point M. T. Nov. 6	$18^h\ 06^m\ 52^s.78$	

15. To find the mean time of the sun's or a star's transit.

The time of the sun's transit is apparent noon (see Art. 4) The corresponding mean time will be found as in Art. 11.

The sidereal time of a star's transit is equal to its right ascension. It is converted into mean time as in Art. 14.

Example.—To find the mean time of transit of α Aquilae (Altair) at St. Hyacinthe, P. Q., on Sept. 18, 1877.

Longitude $4^h\ 52^m$. W. (1)

				Var. in	
S. T. on Sept. 11 at 0^h M. T. Gr.	11^h	50^m	02^s	24^h	$3^m\ 56^s.6$
Correction for $4^h\ 52^m$			48	4^h	39.4
S. T. at 0^h M. T. St-Hyacinthe	11	50	50	40^m	6.6
Altair's Right Ascension	19	44	50	10	1.6
Sidereal interval from mean noon	7	54	00	2	.3
				$4^h\ 52^m$	$47^s.9$

By table "For converting sidereal into mean time intervals

7^h S. T. =	6	58	51.2 M. T.
54^m =		53	51.2
M. T. of transit at St-Hyacinthe, Sept. 18	7^h	52^m	42^s

(1) In this example, fractions of a second are not taken into account.

16. Lower Transits.

Stars near the pole have, at least, two meridian transits above the horizon every day, one above and the other below the pole. The time of upper transit is found as explained; the lower transit takes place 12 sideral hours or $11^h\ 58^m\ 02^s$ M. T. after or before the upper transit. The calculation may be performed as above, by adding 12 hours to the star's right ascension.

The transits occuring at intervals of 24^h S. T. or $23^h\ 56^m\ 04^s$ M. T. there may be two upper or two lower transits in the same mean solar day.

Example.—Required the mean time of lower transit of Polaris at Quebec, on Oct. 13, 1877.

Longitude $4^h\ 45^m$ W.

				Var. in	
S.T. on Oct. 13 at 0^h M.T. Gr.	13^h	28^m	$36^s.2$	24^h	$3^m\ 56^s.6$
Correction for $4^h\ 45^m$			46 .8		
S. T. at 0^h M. T. Quebec	13	29	23 .0	4^h	39 .4
12^h + Polaris' Right Ascension (1)	37	14	32 .7	40^m	6 .6
				5	0 .8
Sidereal interval from mean noon	23	45	09 .7	$4^h\ 45^m$	$46^s.8$
By table for converting sidereal into mean time intervals					
23^h S. T =	22	56	13 .9		
45^m		44	52 .6		
$9^s.7$			9 .7		
Mean time of lower transit Oct. 13	23^h	41^m	$16^s.2$		

(1) 24^h are added to that number, in this case, to make the subtraction possible.

17. Solar and Stellar observations, Methods of observing.

We will suppose the instrument used to be a transit theodolite (1). This being properly adjusted, if a star is to be observed, the telescope is directed to it, and the intersection of the wires fixed precisely upon it. The time of the clock or watch is noted, if required, at the instant when the star appears to be exactly covered by the intersection of the wires. Having read the verniers, the telescope will be turned over, and directed again to the star by turning the vernier plate through 180°, when the same operation as above is repeated.

The field of the telescope must be illuminated to render the wires visible at night. Transit theodolites are generally provided with means for illuminating. If not, a light may be held in front of the object glass, but this method is very imperfect.

For solar observations, a colored glass of a suitable tint is adjusted to the eye piece and the intersection of the wires directed to the centre of the sun. This is only an approximate method; if more precision is required, the limbs, instead of the centre, must be placed on the wires. In this case, the limbs to be observed with the telescope reversed, are the opposite to those in the first position, so that the mean of the results is equivalent to an observation of the sun's centre.

When both altitude and azimuth observations are being made, and the limbs sighted, the image of the sun is placed so that it leaves one of the wires, while it is advancing towards the other. Suppose, in Fig 1, the apparent direction of the sun's motion to be represented by

(1) A simple transit may be used, however, when altitudes are not required.

the arrow, and the wires by AD and BC, Clamp the telescope so that the image of the sun is in *ss'*, and keep AD tangent to the disc by moving the tangent screw of

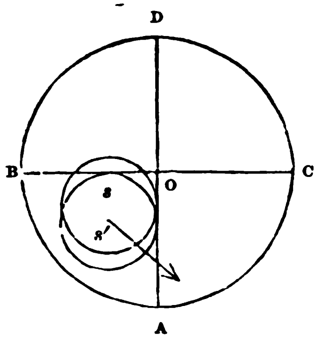

Fig. 1.

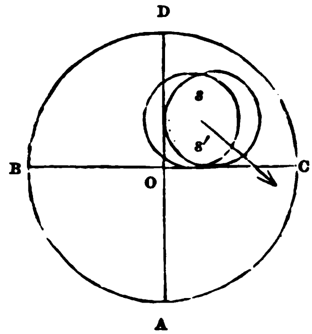

Fig. 2.

the horizontal vernier plate, that of the vertical circle remaining clamped. The sun will appear to move only vertically, and at a certain time, it will be tangent to both wires, at which time the observation is taken.

In the reversed position, Fig 2, the image is placed in *ss'* in the angle DOC opposite to AOB, where it was in the first or direct position. The horizontal wire is kept tangent to the limb by means of the tangent screw of the vertical circle, till the disc is tangent to the vertical wire.

The object of this method is to have to move only one tangent screw at one time, as it would be difficult to make a good observation while moving two screws.

If the sun's motion be different from that shewn in the figure, the disc's image should always be so placed, in relation to the wires, that it is advancing toward one wire while leaving the other; the wire towards which it is advancing is the one to be moved by the tangent screw, because, the only visible part of the wire being that pro-

jected on the disc, it is more easily seen when it leaves than when it comes upon the limb.

When no colored glasses are provided, the telescope is adjusted for focus on a distant object, and the eye piece drawn out a little. Directing it to the sun, the images of the disc and wires should be made fall on a white screen, such as a sheet of paper, held behind the eye piece. The observation is then made as before, when the sun is seen tangent to the wires, projected on the sheet of paper.

The distance of the screen is adjusted by moving it forward and backward until the image is sharply defined. This distance, and the size of the image are decreased when the eye piece is drawn out and increased when pushed in.

The telescope is readily directed to the sun by revolving the instrument till the shadow of the vertical circle is in its own plane, and turning the telescope up or down till its shadow forms a circle. The sun is then in the field.

The vertical circle reading is not always the altitude; sometimes it is the zenith distance, sometimes that distance or the altitude plus 90°. The altitude is the difference between the reading when the telescope is horizontal and the reading when directed to the sun or star. Should the number, used as reading with the telescope horizontal, not be accurate there will be an error in the altitude arising from it; but when the observation is made in both positions of the telescope, this error affects both altitudes precisely by the same quantity, but with diffrent signs. It will, therefore, disappear in the mean result.

The altitudes, horizontal circle readings and times employed in the subsequent calculations are supposed to

be always the mean of two observations, made as described above, in both positions of the telescope. Besides the advantages already alluded to, the results are independent of certain instrumental errors, such as collimation and inclination of horizontal axis.

18. Correction of altitudes.

For altitudes of the sun, subtract the refraction (Table I) and add the parallax in altitude (Table II). If the observation was not taken as directed in Art. 17, there might be a correction for semidiameter which we will not consider it here.

When great accuracy is not required, the correction for parallax may be dispensed with. The altitude's correction is then the same as for a star: refraction only.

Example 1.—The apparent altitude of the sun being 21° 07′ 15″, find the true altitude.

Apparent altitude	21° 07′ 15″
Refraction (Table I)	2 31
Difference	21 04 44
Parallax (Table II)	8
True altitude	21° 04′ 52″

Example 2.—The apparent altitude of Polaris being 47° 46′ 22″ find the true altitude.

Apparent altitude	47° 46′ 22″
Refraction (Table I)	53
True altitude	47° 45′ 29″

CHAPTER II.

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TIME

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19. Definitions.

The zenith distance is the complement of the altitude, therefore it is found by subtracting the altitude from 90°.

The polar distance is the complement to 90° of the declination when the heavenly body is north of the equator, or 90° plus the declination, when south. In the southern hemisphere, the contrary would be the case. At the head of the Nautical Almanac columns are the letters N or S or the words North or South, which denote on what side of the equator the heavenly body is.

The colatitude is the complement to 90° of the latitude.

A cologarithm is a logarithm subtracted from 10. It is found by subtracting each figure from 9 except the last one which is subtracted from 10. With a little practice, they can be read from the ordinary logarithmic tables almost as easily as the logarithm itself. By their use, subtraction is performed by addition, as adding the cologarithm or subtracting the logarithm will produce results differing exactly by 10 in the characteristic.

We may remark that:

Cologarithm sine *a* is the same	as	Logarithm	cosecant *a*
" cosine *a*	"		secant *a*
Tangent *a*	"		cotangent *a*

and they may be used indifferently (1)

20. To find the time by the sun's meridian transit.

A little before noon, the telescope of the instrument should be directed to astronomical south, and placed at the proper altitude. When the sun's centre is behind the vertical wire, it is apparent noon (Art. 4.). The corresponding mean time will be found as in Art. 11 and 15. It is more accurate to note the times of transit of the East and West limbs and take the mean, which will correspond to the sun's centre, or to apparent noon.

When there is no colored glass, the observation is made on a screen, as explained in Art. 17.

21. To find the time by a star's meridian transit.

Having fixed the telescope to astronomical south, when the star passes the vertical wire, the time of the clock or watch is noted. The mean time of transit is deduced as in Art. 15 and 16. The stars south of the zenith will give more reliable results than the stars to the north of it, because they move quicker.

---

(1) Some tables, such as Caillet's, Gauthier Villars, Paris, contain the logarithms and cologarithms. They also give the logarithms of trigonometric lines for angles expressed in arc and in time, which is often very convenient.

22. To find the name of an unknown star passing the meridian.

When none of the stars near the meridian are known, the brightest will be observed, and the time of transit and altitude noted. If the clock or watch is not far astray, the name of the star will be found thus:

For stars south of the zenith,—Find the difference between the star's zenith distance and the latitude, it is the approximate declination, South, if the zenith distance is greater than the latitude and North if it is less.

To the sideral time at Greenwhich mean noon (from the Nautical Almanac) add the mean time of observation, the result is the approximate right ascension.

For stars between the pole and zenith.—Add the zenith distance to the latitude, it will give the approximate northern declination.

The approximate right ascension is found as above.

For stars below the pole.—Add the altitude to the colatitude to obtain the approximate Northern declination.

Find the approximate right ascension in the same manner as above, adding 12 hours to the result.

See then in the Nautical Almanac which star has the corresponding declination and right ascension; it is the star observed. If there be no star in that position, search should be made amongst the planets.

Example.—At St. Hyacinthe, P. Q., on Sept. 18, 1877, the meridian transit of a first magnitude star was observed at 52° 50′ altitude South; the time shewn by the watch's face was $7^h\ 52^m$ P. M., required its error.

Latitude 45° 39′ N. Longitude $4^h\ 52^m$ W.

St-Hyacinthe M. T. Sept. 18	$7^h\ 52^m$
S. T. on Sept. 18 at 0^h M. T. Gr.	11 50
Approximate right ascension	$19^h\ 42^m$
Altitude of Star	52° 50′
Zenith distance	37 10
Latitude St-Hyacinthe	45 39
Approximate declination	8° 29′ N.

In the Nautical Almanac list, we find that the above right ascension and declination correspond to α Aquilae (Altair).

The calculation being carried on as in the example of Art. 15, we find:

M. T. of Altair's transit	$7^h\ 52^m\ 42^s.4$
Watch time	7 52 00
Watch slow	$42^s.4$

23. To find the time by the sun's altitude.

To obtain accurate result, the observations should be made not less than three hours before or after noon.

Observe the sun's altitude (Art. 17), correct it (Art. 18) and find the polar distance of the sun (Art. 19). Add together the true altitude, the latitude and the polar distance, take half the sum, and subtract the altitude from it. Find the cologarithm (Art. 19) cosine latitude, the cologarithm sine polar distance, the logarithm cosine half sum, and the logarithm sine half sum minus the altitude. Add and take half the total, you will have the logarithm sine of half the sun's hour angle. This being taken out of the table, and converted to time (Art. 2)

gives the apparent time if the sun was West of the meridian ; if East, the hour angle must be subtracted from 24^h. The mean time is deduced as in Art. 11. (1)

Example 1.—On Aug. 1, 1877 at Montreal, the following observations were taken in the manner explained in Art. 16:

Position of telescope	Sun's altitude	Watch time
Direct	25° 56′	$4^h\ 48^m\ 06^s$ P. M.
Reversed	25 32	4 49 02

Required the watch error on mean time
Latitude 45° 29′ N. Longitude $4^h\ 54^m$ W. (2)

Montreal approximate M. T. Aug. 1,	$4^h\ 49^m$
Longitude	4 54 W
Greenwich approximate M. T. Aug. 1,	9 43 $= 9^h, 7$

---

(1) The formula is:

$$\sin \tfrac{1}{2} P = \sqrt{\frac{\cos S \sin (S - A)}{\cos L \sin \Delta}}$$

$$\text{where } S = \frac{A + L + \Delta}{2}$$

A = True altitude.

L = Latitude.

Δ = Polar distance.

P = Sun's hour angle.

(2) In this example, seconds of arc and fractions of a second of time are not taken into account.

Polar Distance.

Sun's declination on Aug. 1 at 0^h M. T. Gr.	17°	57′ N
Correction for $9^h\ 43^m$		6
Sun's declination on Aug. 1 at $4^h\ 49^m$ Montreal M. T.	17	51 N
Polar distance	72	09

Var. in 1^h	38″.02
Multiplied by	9 . 7
	26614
	34218
Var. in $9^h\ 43^m$	368″.794

Equation of Time.

Equation of time on Aug. 1 at 0^h M. T. Gr. (To be added to A. T.)	$6^m\ 02^s$
Correction for $9^h\ 43^m$	1
Equation of time on Aug. 1 at $4^h\ 49^m$ Montreal M. T.	$6^m\ 01^s$

Var. in 1^h	$0^s.15$
Multiplied by	9 . 7
	105
	135
Var. in $9^h\ 43^m$	$1^s.455$

Mean of the sun's apparent altitudes	25°	44′		
Refraction minus parallax		2		
True altitude	25	42		
Latitude	45	29	colog. cos	0.15421
Polar distance	72	09	colog. sin.	0.02143
Sum	143	20		
Half sum	71	40	log. cos.	9.49768
Half sum minus altitude	45	58	log. sin.	9.85669
Sum				19.53001

Half sum or log. sin. half the hour angle	9.76500
Half the hour angle	35° 36′
" in time	$2^h\ 22^m\ 24^s$
Hour angle or apparent time	4 44 48
Equation of time (to be added to A. T.)	6 01
M. T. of observation	4 50 49
Mean of the watch times	4 48 34
Watch slow	$2^m\ 15^s$

Example 2.—The following observations were taken at Toronto, on Dec. 18, 1877.

Position of telescope	Sun's altitude	Watch time
Direct	10° 22′ 10″	$8^h\ 57^m\ 11^s$ A. M.
Reversed	10 40 15	8 59 47

Required the watch error on mean time

Latitude 43° 39′ 26″ N. Longitude $5^h\ 18^m$ W.

Toronto Approximate M. T.	Dec. 17,	$20^h\ 58^m$
Longitude		5 18 W
Greenwich approximate M. T.	Dec. 18,	$2^h\ 16^m = 2^h, 3$

Polar Distance.

Sun's declination on Dec. 18 at 0^h M. T. Gr.	23° 25′ 00″ S	Var. in 1^h 3″.8
		Multiplied by 2 .3
Correction for $2^h\ 16^m$	9	11.4
		76
Sun's declination on Dec. 17 at 20^h 58^m M.T. Toronto	23 25 09 S	
Polar distance	113 25 09	Var. in $2^h\ 16^m$ = 8″.74

Equation of Time.

Eq. of time on Dec. 18 at 0^h M. T. Gr. (to be subtracted from A.T.)	3^h 01^m 6	Var. in 1^h	$1^s.24$
Correction for 2^h 16^m	2 9	Multiplied by	2 . 3
Eq. of time on Dec. 17 at 20^h 58^m M.T. Toronto	2^m 58^s 7		372
			248
		Var. in 2^h 16^m =	$2^s.852$

Mean of the sun's apparent altitudes	10°	31′	13″		
Refraction		5	05		
Difference	10	26	08		
Parallax			9		
True altitude	10	26	19		
Latitude	43	39	26	colog. cos	0 14058 (1)
Polar distance	113	25	09	colog. sin	0 03734
Sum	167	30	52		
Half sum	83	45	26	log. cos	9.03632
Half sum minus the altitude	73	19	09	log. sin	9.98133

Sum			19.19557
Half or log. sin. half the hour angle			9.59779
Half the hour angle	23°	20′	00″
" in time	1^h	33^m	$20^s.0$
Hour angle or 24^h minus apparent time	3	06	40 .0
Apparent time	20	53	20 .0
Eq. of time (to be subtracted from A.T.)		2	58 .7
M. T. of observation	20	50	21 .3
Mean of the watch times	20	58	29 .3
Watch fast		8^m	$07^s.7$

(1) The logarithms, in this example, are only taken to the nearest 15″

24. To find the time by the altitude of a star.

To obtain an accurate result, the observation should be made on a star as much East or West as possible.

The calculation is the same as for altitudes of the sun, but the result, instead of being apparent time, is the star's time, to which must be added the right ascension to obtain the sidereal time. This is converted into mean time as in Art. 14.

Example. On Sept. 28, 1877, at Three Rivers, P. Q. the following altitudes of β Orionis (Rigel), were taken, the star being East of the meridian.

Position of telescope	Star's altitude	Watch time
Direct	21° 30′	$7^h\ 30^m\ 05^s$ P. M.
Reversed	21 44	7 31 33

Required the watch error on mean time.

Latitude 46° 20 N. Longitude $4^h\ 50^m$ W. (1)

Sidereal Time.

S. T. on Sept. 28 at 0^h M. T. Gr.	$18^h\ 28^m\ 14^s\ 5$	Var. in 24^h	$3^m\ 56^s.6$
Cor. for $4^h\ 50^m$	47 6	4	39 .4
S. T. at 0^h M. T. Three Rivers	$18^h\ 29^m\ 02^s\ 1$	40^m	6 .6
		10	1 .6
		Var. in $4^h\ 50^m$	$47^s.6$

(1) In this example, quantities less than 1 minute of arc or 1 second of time, are not taken into account.

Mean of the star's altitudes	21°	37′		
Refraction		2		
True altitude	21	35		
Latitude	46	20	colog. cos.	0.16086
Polar distance of the star	98	21	colog. sin.	0.00463
Sum	166	16		
Half sum	83	08	log. cos.	9.07758
Half sum minus altitude	61	33	log. sin.	9.94410

Sum	19.18717		
Half sum or log. sine half the hour angle	9.59358		
Half the hour angle	23°	5′	45″
" in time	1^h	32^m	23^s
Hour angle or 24^h minus the star's time	3	04	46
Star's time	20	55	14
Star's right ascension	5	08	42
S. T. of observation	26	03	56
S. T. at 0^h M. T. Three Rivers	18	29	02
Sidereal interval from mean-noon	7	34	54

By table " for converting sidereal into mean time intervals "

7^h S.T. =	6	58	51.2
34^m		33	54.4
54^s			53.9

M. T. of observation	7	33	40
Mean of the watch times	7	30	49
Watch slow		2^m	51^s

25. Graphic method for finding the time by altitudes of the sun or a star.

Describe a circle. Through the centre O (Fig. 3 and 4) draw two lines forming an angle EOH equal to the colatitude (Art. 19). Make HOB equal to the altitude and EOA to the declination, A being to the left of E if the declination is North, and to the right if South. From A, let fall a perpendicular A*c* on EO and with O as centre and O*c* as radius, describe a circle. Through A and B, draw two lines A*s* and B*s* parallel to HO and EO. At *s* erect a perpendicular *ss'* to A*s*, on the right hand side, if the sun or star be in the West or on the left,

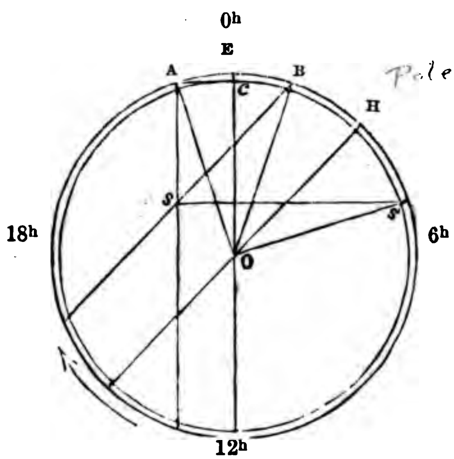

Fig. 3.

if in the East. Join O to the intersection point of *ss'* with the inner circle, and the angle EO*s'*, reckoned in the direction of the arrow, will be the apparent or star's time, according as the sun or a star was observed. The mean time is deduced as in Art. 23 and 24.

Fig. 3 is the solution of Example 1, Art. 23, HOE is the colatitude, 44° 31′, AOE is the sun's declination, 17° 51′ North, BOH the true altitude, 25° 42′ and EO*s*′, which is equal to 71° 12′ or 4^h 44^m 48^s is the apparent time.

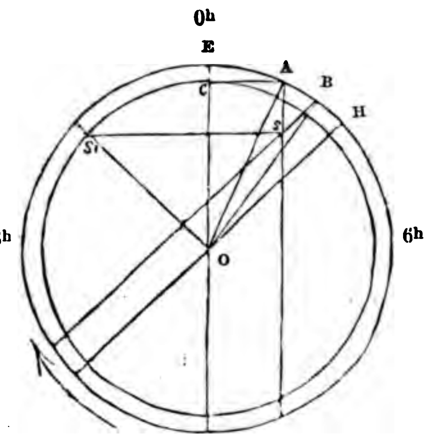

Fig. 4.

Fig. 4 is the solution of Example 2, Art. 23, HOE is Toronto colatitude, 46° 21′, AOE the sun's declination 23° 25′ South, BOH the true altitude 10° 26′ and EO*s*, which is equal to 313° 20′ or 20^h 53^m 20^s is the apparent time.

These figures should be made on a much larger scale, and the angles platted by means of a scale or table of chords.

CHAPTER III.

LATITUDE.

26. Latitude by the Meridian altitude of the Sun.

When the sun is near the meridian, its motion in altitude is very small. If one of the limbs be observed a little before apparent noon and the other a little after, the mean of the altitudes may be assumed to be, within the limits of precision of this work, the meridian altitude of the sun's centre. The observation should be made with the telescope in both positions, direct and reversed.

Find, then, the true altitude (Art. 18). Add the sun's polar distance, and subtract from 180°. The result is the latitude.

Example.—On May 3, 1878, at Chicoutimi, P. Q., the altitudes of the sun on the meridian were:

with telescope direct, lower limb	57° 09′ 39″
" reversed, upper limb	57 41 25

Required the latitude of the place.

Longitude of Chicoutimi, $4^h\ 42^m$ W

Sun's declination on May 3 at 0^h A. T. Gr	15° 47′ 21″ N	Var. in 1 hour 43″ 85
Correct. for 4^h 42^m	3 26	Multiplied by 4.7
		30695
		17540
Sun's declination on May 3 at 0^h A. T. Chicout.	15 50 47 N	V. in 4^h 42^m=206″.095
Polar distance	74° 09′ 13″	

Mean of the sun's apparent altitudes	57° 25′ 32″
Refraction	37
Difference	57 24 55
Parallax	4
True altitude	57 24 59
Polar distance	74 09 13
Sum	131 34 12
Latitude	48° 25′ 48″ N

27. Latitude by the meridian altitude of a star.

For stars south of zenith, make the same calculation as for the sun,

For stars between the zenith and pole, subtract the polar distance from the true altitude,

For stars below the pole, add the polar distance to the true altitude. (1)

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(1) The formulae are:

For stars passing the meridian south of zenith $L=180^\circ-(A+\Delta)$

" between zenith and pole $L=A-\Delta$

" below the pole $L=A+\Delta$

Where L is the latitude, A the altitude and Δ the polar distance.

If the star be unknown, its name would be found in the manner indicated at Art. 22.

Example.—On Jan. 6, 1877, at Metabetchouan, (Lake St. John) P. Q. the altitude of the pole star at upper transit was 49° 46′ 50″. Required the latitude.

Declination on Jan. 6, 1877	88° 39′ 37″
Polar distance	1 20 23
Apparent altitude	49° 46′ 50″
Refraction	49
True altitude	49 46 01
Polar distance	1 20 23
Latitude	48° 25′ 38″ N

28. Latitude by the altitude of the pole star at any time.

It is requisite to have a watch, whose error on mean or sidereal time has been determined (Chapter II), and to note the time of observation. The latitude will be calculated by means of the tables at the end of the Nautical Almanac "used in determining the latitude by ob" servations of the pole star out of the meridian."

The method of using them is as follows:

Find the true altitude and take 1′ from it. (1)

From the watch time, deduce the sidereal time (Art. 13)

With that time, take out the "first correction." It must be added to the true altitude if the sign be + and subtracted if —.

(1) This correction might possibly be dispensed with in subsequent editions of the Nautical Almanac. The explanation given at the end of it, should be read before using the tables.

To the result, add the "second" and "third corrections"; the sum is the latitude.

The error in the latitude for one minute of time never exceeds 21″.

Example.—On July 25, 1877, at Cape Rosier Light, Gaspé, P. Q., the altitude of the pole star was 47° 48′ 53″ at $7^h\ 33^m\ 12^s$ P. M. Required the latitude of the light.

Longitude $4^h\ 17^m$ W.

		Var. in	
S. T. on July 25, at 0^h M. T. Gr.	$8^h\ 13^m\ 11^s\ 8$	24^h	$3^m\ 56^s\ 6$
Correction for $4^h\ 17^m$	42 2	4^h	39 4
		15^m	2 5
S. T. on July 25, at 0^h M. T. Cape Rosier	8 13 54 0	2^m	3
By table for converting mean into sidereal intervals:		$4^h\ 17^m$	42 2
7^h M. T. =	7 01 09 0 S. T.		
33^m	33 05 4		
12^s	12 0		
S. T. of observation	$15^h\ 48^m\ 20^s\ 4$		

Apparent altitude		47°	48′	53″
Refraction				53
True altitude		47	48	00
Minus			1	00
		47	47	00
First correction	+	1	03	06
		48	50	06
Second correction				28
Third correction			1	03
Latitude		48	51	37 N

CHAPTER IV.

AZIMUTH.

29. Definition.

The azimuth of a line or object is the horizontal angle formed by the meridian and the direction of the line or object. It is astronomical or magnetic according as the meridian is astronomical or magnetic.

In the present work, the azimuths are reckoned from 0° to 360° from the North through East, South and West, 0° being North, 90° East, 180° South and 270° West. Thus 275° will be written instead of N 85° W, 125° instead of S 55° E and 190° instead of S 10° W. This arrangement is devised in order to apply to instruments whose horizontal circle is graduated from left to right as in Fig. 5. Unfortunately, many instruments are graduated differently; but it is so simple and easy to alter the methods given, and to adapt them to any instrument, that it was thought better to use, in this work, only the circle divided as in Fig. 5, so as to avoid confusion in the explanations, examples and tables. For instance, in the

following chapters and tables, 360° minus the azimuth is to be substituted to the word azimuth, when the instrument used is graduated from right to left.

In observing horizontal angles, the lower circle is clamped; the telescope, then, is turned with the vernier plate, directed on each point to be observed, and the corresponding readings of the vernier noted. In the subsequent calculations, only these *readings* and not the *angles* between the objects, are used.

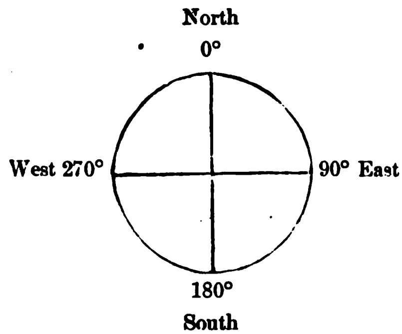


Fig. 5.

30. To deduce, from an observation, the astronomical azimuth of a line or object.

In both positions of the telescope, direct and reversed, point the telescope to the sun or star and to the line or object, and note the corresponding horizontal circle readings. Having found the azimuth of the sun or star by one of the methods of the present chapter, subtract from it the mean of the horizontal circle readings on the sun or star, and add the mean of the readings on the line or

object. The result will be the required azimuth. 360° must be added when the subtraction is not possible, and subtracted when the result is greater than 360°.

Example.—The following observations were made at Tadousac, P. Q.:

Position of telescope.	H. C. R. on Lark Island light.	H. C. R. on the sun.
Direct	34° 40′ 10″	344° 23′ 00″
Reversed (1)	34 39 50	344 35 00

The calculated azimuth of the sun was 101° 12′ 00″.

Required the azimuth of Lark Island light.

Azimuth of the sun	101° 12′ 00″
Mean of the H. C. R. on sun	344 29 00
Difference	116 43 00
Mean of the H. C. R. on Lark Id. light	34 40 00
Azimuth of Lark Id. light	151° 23′ 00

31. Azimuth by the meridian transit of the pole star.

The upper transit of Polaris takes place 24 minutes after its passage in the same vertical plane with ε Ursae majoris (Alioth). This interval, 24 minutes, increases yearly by 17 seconds.

The lower transit occurs 26 minutes after the passage of

(1) In reality, these readings were 214° 39′ 50″ and 164° 23′ 00″, but for simplicity, it is convenient to add 180° to the H. C. R. in the position reversed.

Polaris in the same vertical plane with γ Cassiopeiae. This interval, 26 minutes, increases yearly by 19 seconds. (1)

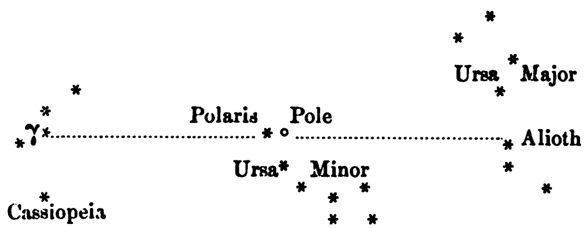


Fig. 6.

Both stars are about on the same line with the pole star (Fig. 6.) They afford a very simple method of determining an azimuth. The instrument being adjusted, and the horizontal circle clamped, direct the telescope to the pole star, clamp the vernier plate and lower the telescope to the altitude of " Alioth " or " γ Cassiopeiae." Repeat the same operation till the lower star crosses the vertical wire, note the time, and 24 or 26 minutes after, the pole star will be on the meridian.

It is more accurate to find the time of transit, (Art. 15 and 16) and observe it with a watch whose error has been determined by one of the methods of Chapter II.

A very reliable result will be obtained by observing the pole star at the same interval of time after as before the transit. The process is as follows, the H. C. R. being noted in each case.

1°. Direct the telescope to the object.

2°. A certain time before transit, say 2 minutes, bring the vertical wire on Polaris.

(1) Calculated for 1878 and Latitude 47°

3°. Reverse, and 2 minutes after transit, repeat the observation on Polaris.

4°. Sight to the object.

The mean of the readings on Polaris corresponds to the meridian.

In Canada, an error of one minute in the time affects the azimuth by about 30″.

Example.—At Tadousac, P. Q., the following observations were made, the pole star being observed 2 minutes before and 2 minutes after meridian transit:

Position of telescope.	H. C. R. on Black Pt. light.	H. C. R. on Polaries.
Direct	191° 14′ 05″	3° 36′ 00″
Reversed (1)	191 13 55	3 38 00

Required the azimuth of Black Pt. light.

Azimuth of Polaris	360° 00′ 00″
Mean of the H. C. R. on Polaris	3 37 00
Difference	356 23 00
Mean of the H. C. R. on Black Pt. light	191 14 00
Azimuth of Black Pt. light	547 37 00
or	187° 37′ 00″

31. Azimuth by the elongation of the pole star.

This method is very accurate, and is always employed when great precision is aimed at.

Table III gives the approximate times of elongation. A little before that time, the distance of the star from

(1) See note of Art. 30.

the meridian is increasing, then it moves only in altitude, and after elongation, it begins to move in the opposite direction in regard with the vertical wire.

Table IV gives the azimuth at Eastern elongation, from which the other is readily deduced. It is calculated with the mean positions of Polaris for the latitudes 42° to 54° and the years 1878 to 1890.

When more accuracy is required, add the logarithm cosine declination to the cologarithm cosine latitude, the result is the logarithm sine of the azimuth. When the star is at Eastern elongation, the azimuth is the smallest angle found in the table, corresponding to the logarithm sine; at Western elongation, it is 360° minus that angle. (1)

Example.—On June 20, 1878, at Trois-Pistoles, P. Q., the H. C. R. on the pole star at Western elongation was 339° 41′, and on Black Point light (Saguenay) 251° 18′. Find the azimuth of light.

Latitude of Trois-Pistoles 48° 08′ N.

1°. *By Table IV.*

Azimuth of Polaris at Eastern elongation	2°	00′	30″
" Western "	357	59	30
H. C. R. on Polaris	339	41	00
Difference	18	18	30
H. C. R. on Black Pt. light	251	18	00
Azimuth of Black Pt. light	269°	36′	30″

(1) The formula is: $\sin Z = \frac{\cos D}{\cos L}$, where Z is the azimuth, D the declination of Polaris and L the latitude.

2°. By Logarithms.

Declination of Polaris on Jan. 20, 1878, 88° 39′ 58″ N.

Log. cos. declination	8.36696
Colog. cos. latitude	0.17561
Log. sine azimuth	8.54257
360° minus azimuth	1° 59′ 56″
Azimuth	358 00 04
H. C. R. on Polaris	339 41 00
Difference	18 19 04
H. C. R. on light	251 18 00
Azimuth of Black Pt. light	269° 37′ 04″

33. Azimuth by observation of the pole star at any time.

It is requisite to have a watch whose error has been determined (see Chapter II.) The observation is made in both positions of the telescope, and the altitudes, H. C. R., and times of observation noted. The sidereal time of observation is deduced from the time of the watch by the method of Art. 13, and the right ascension of Polaris subtracted from it. When necessary, 24 hours are added to make the subtraction possible. Convert, then, the remainder into arc (Art. 3,) add its log. sine to the log. cosine of the declination and the colog. cosine of the altitude (1). The sum is the sine of the azimuth. The star is East of the meridian when the difference between

(1) The apparent altitude may be used.

the sidereal time and right ascension is greater than 12 hours, and West when less. Therefore, in the first case the azimuth is the smallest angle corresponding, in the table to the log. sine azimuth, and in the other case it is 360° minus that angle (1).

Example 1.—The following observations were taken on May 26, 1877, on the exterior line of township Ashuamp-mouchouan. (Lake St. John) P. Q.

Position of Telescope.	Time of the watch.	H. C. R on a light placed in the line.	H. C. R. on Polaris.	Altitude of Polaris.
Direct	$7^h\ 04^m\ 15^s$ P.M.	233° 41′ 40″	359° 46′ 30″	47° 18′
Revers. (2)	7 06 09	233 41 20	359 47 30	47 20

The watch was 54^s slow, and the longitude of the place $4^h\ 50^m$ W. Required the azimuth of the line.

Mean of the watch times	$7^h\ 05^m\ 12^s$ P. M
Watch slow	54
M. T. of observation May 26	7 06 06

(1) This is only an approximate method. The formulae are :

$$\sin Z = \frac{\sin T \cos D}{\cos A}$$

and $T = S - R$

where Z is the azimuth, D the declination, A the altitude, S, the sidereal time of observation and R the right ascension.

(2) See note of Art. 30.

S. T. on May 26, at 0^h M. T. Gr.	$4^h\ 16^m\ 38^s$	Var. in 24^h	$3^m\ 57^s$
Correction for $4^h\ 50^m$	48	4	39
S. T. on May 26, at 0^h M. T. Ashuamp.	4 17 26	40^m	7
		10^m	2
By table for converting mean into sidereal intervals,		$4^h\ 50^m$	48

7^h M. T. =	7 01 08 S. T.
6^m	6 01
6^s	6
Sidereal time of observ.	11 24 41
Right ascension of Polaris	1 12 53
Difference	$10^h\ 11^m\ 48^s = 152°\ 57'$

Log. sine 152° 57	9.6579
Log. cosine declination (88° 39′ 09″)	8.3716
Colog. cosine altitude (47° 19′)	0.1688
Log. sine azimuth of Polaris	18.1983
360° minus azimuth of Polaris	0° 54′ 17″
Azimuth of Polaris	359 05 43
Mean H. C. R. on Polaris	359 47 00
Difference	359 18 43
Mean H. C. R. on light	233 41 30
Azimuth of the line	593 00 13
or	233 00 13

Example 2.—The following observations were taken at Halifax, N. S., on November 20, 1877:

Position of telescope.	Watch time.	H. C. R. on Polaris.	H. C. R. on a Reference light.	Altit. of Polaris.
Direct	$8^h\ 25^m\ 03^s$ P.M.	0° 22′ 30″	359° 14′ 40″	45° 52′
Revers. (1)	8 27 23	0 21 40	359 14 30	45 53

The watch was $3^m\ 15^s$ slow and the longitude of the place $4^h\ 14^m$ W. Required the azimuth of the reference light.

Mean of the watch times	$8^h\ 26^m\ 13^s$ P. M.
Watch slow	3 15
Halifax M.T. of observation Nov. 20,	8 29 28

S. T. on Nov. 20 at 0^h M. T. Gr.	$15^h\ 58^m\ 25^s$	Var. in 24^h	$3^m\ 57^s$
Var. in $4^h\ 14^m$	41	4	39
S. T. on Nov. 20 at 0^h M. T. Halifax	15 59 06	14	2
		V. in $4^h\ 14^m$	41^s

By table for converting mean into sidereal intervals

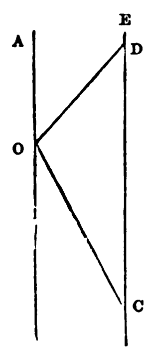
	8^h M. T =	8 01 19 S. T.
	29^m	29 05
	28^s	28
S. T. observation		24 29 58
Polaris Right ascension		25 14 27
Difference		$44^m\ 29^s$ = 11° 07′ 15″

(1) See note of Art. 30.

Log. sine 11° 07′ 15″	9.2853
Log. cosine declination (88° 39′ 48″)	8.3679
Colog. cosine altitude (45° 52′ 30″)	0.1572
Log. sine Azimuth of Polaris	17.8104
Azimuth of Polaris	0° 22′ 13″
Mean H. C. R. on Polaris	0 22 05
Difference	0 00 08
Mean H. C. R. on light	359 14 35
Azimuth of reference light	359° 14′ 43″

34. Graphic solution of the preceding problem.

Draw OC (Fig. 7) equal to the polar distance of the star, upon a scale of a convenient number of minutes to the inch. Make the angle AOC equal to the difference between the sidereal time and right ascension. Through C, draw CE parallel to AO, and make AOD equal to the zenith distance of the star. OD is, by the scale of the plan, the azimuth or 360° minus it, according as the star is East or West of the meridian.



Scale: 80′ to the inch.

Fig. 7

Fig. 7 is the solution of Example 1, Art. 33. OC is equal to the polar distance 80′ 12″ at the scale of 80′ to the inch, AOC is equal to $10^h\ 11^m\ 48^s$ or 152° 57′, AOD is the zenith distance 42° 41′ and OD, measured with the scale of 80 to the inch gives 54′ for the complement to 360° of the azimuth.

35. Azimuth by observation of Polaris when vertical with other stars.

Selection of stars.—To the sidereal time at Greenwich mean noon, add the mean time best suited for your observation. The result is the approximate corresponding sidereal time. In Table V, the star will be chosen, whose sidereal time of transit over the vertical of Polaris, given at the head of the column, is nearest to the sidereal time found.

To find the mean time when the selected star is vertical with Polaris.—It is necessary to know it approximately, in order to be ready for observation. From the sidereal time when vertical with Polaris, (Table V) substract the sidereal time at Greenwich mean noon, the difference is, within a few minutes, the required mean time. 24 hours are added, when necessary, to make the subtraction posible.

Observation.—About 10 minutes before the time found, direct the telescope of the instrument to Polaris and clamp the horizontal circle and vernier plate. Fix the telescope to the altitude given in Table V for the star and the latitude of the place; its direction will then be so nearly towards the star as to cause no difficulty in identifying it, those given in Table V being the brightest of that part of the firmament in which they are situated. Wait till the star is vertical with Polaris, which will be seen by moving the telescope round the horizontal axis, from one star to the other. Fix it, then, on the pole star, whose azimuth for that instant is given in Table V.

This table is calculated with the mean positions of stars for 1878. For those North of the zenith, the transit below the pole is the only one taken into consideration.

A result more accurate can be obtained by calculating the azimuth with the apparent positions of stars for the day of observation. The log. sine of the azimuth is found by adding together the log. sine of the difference of the right ascensions, the log. cosines of the declinations, the colog. cosine of the latitude and the colog. sine of the distance from Polaris, which latter is given in Table V. It will be seen, by the table, wether the azimuth is to be taken between 270° and 360° or between 0° and 90°. (1)

Example.—On Dec. 6, 1877 the pole star was observed when vertical with α Ursae majoris. The horizontal circle reading on the pole star was 0° 39′ 00″ and on a reference light 233° 36′, required the azimuth of the light.

Latitude 48° 02′

1°. *By Table V.*

Azimuth of Polaris (Table V)	1° 06′
H. C. R. on Polaris	0 39
Difference	0 27
H. C. R. on reference light	233 36
Azimuth of reference light	234° 03′

---

(1) The formulae are : $\sin Z = \frac{\sin T \cos D \cos D'}{\sin d \cos L}$

and $T = R - R'$

where Z is the azimuth, D and D′ the declinations, R and R′ the right ascensions of Polaris and the other star, *d* their distance, and L the latitude.

2°. *By Logarithms.*

Right ascension of α Ursae majoris	$10^h\ 56^m\ 13^s$	Decl. 62° 24′ 15″
" of Polaris	1 14 16	" 88 39 52
Difference	$9^h\ 41^m\ 57^s$	
or	145° 29′ 15″	
Log. sine 145° 29′ 15″		9.75327
Log. cosine declination Polaris		8.36745
" α Ursae		9.66580
Colog. sine distance from Polaris (Table V)		0.31848
Colog. cosine latitude		0.17477
Log. sine azimuth		28.27977
Azimuth		1° 05′ 29″
H. C. R. on star		0 39 00
Difference		0 26 29
H. C. R. on light		233 36 00
Azimuth of reference light		234° 02′ 29″

36. Azimuth by altitudes of the sun.

Observe as explained at Art. 17, noting in each position of the telescope, the altitude of the sun and the horizontal circle readings on the sun and object whose azimuth is required.

Find the true altitude (Art. 18) and the sun's polar distance. Add together the altitude, latitude and polar distance; take half the sum and subtract from it the polar distance.

Add together the colog. cosine of the altitude, the colog. cosine of the latitude, the log. cosine half the sum of the altitude, latitude and polar distance, and the log. cosine of half the same sum minus the polar distance. Half the total is the log. cosine of half the sun's azimuth if it was East or of the supplement of the same angle, if West of the meridian.

This method is one of the simplest and most convenient for surveyors. The observation does not require more than 3 or 4 minutes and can be made without any loss of time, as for instance, when the flagman is going forward and the surveyor waiting for him.

The observation, to be reliable, should not be taken less than 3 hours before or after noon, and the sun's altitude not less than 5°. (1)

Example 1.—On July 15, 1877, at 7^h A. M., Pointe Fortune, P. Q., the following observations were taken to find the azimuth of a signal placed at Carillon :

Position of telescope.	H. C. R. on Sun.	H. C. R. on Signal.	Sun's altitude.
Direct	71° 41′	345° 36′	12° 39′
Reversed (2)	71 59	345 36	12 53

Required the azimuth of the signal.

Latitude 45° 33′ N. Longitude 4^h 58^m W (3)

(1) The formulae are:

$$\text{Cos } \tfrac{1}{2} Z = \sqrt{\frac{\cos S \cos (S-\Delta)}{\cos L \cos A}} \text{ and } S = \frac{A + L + \Delta}{2}$$

where Z is the azimuth, A the true altitude, Δ the polar distance, and L the latitude.

(2) See note of Art. 30.

(3) Arcs less than 1′ are not taken into account in this example.

Pt. Fortune M. T., July 14,		$19^h\ 00^m$
Longitude		4 58 W.
Greenwich M. T., July 14,		$23^h\ 58^m$
Sun's declination on July 15, at 0^h M. T. Gr.		21° 39′ N
Correction for 2^m		0
Sun's decl. on July 14, at 19^h M. T. Pt. Fortune		21 39 N
Polar distance		68° 21′

Mean of the apparent altit.	12° 46′	
Refraction minus parallax	4	
True altitude	12 42	colog. cosine 0.01076
Latitude	45 33	colog. cosine 0.15472
Polar distance	68 21	
Sum	126 36	
Half sum	63 18	log. cosine 9.65256
Half sum minus polar dist.	5 03	log. cosine 9.99831

Sum	19.81635
Half sum or log. cos. half the azimuth	9.90817
Half the azimuth	35° 57′. 5
Azimuth of the sun	71 55
Mean H. C. R. on sun	71 50
Difference	0 05
Mean H. C. R. on signal	345 36
Azimuth of signal	345° 41′

Example 2.—On Oct. 2, 1877, at $3^h\ 20^m$ P. M. Esquimalt, B. C., the following observations were taken :

Position of telescope.	H. C. R. on sun.	H. C. R. on a flag.	Sun's altitude.
Direct	172° 37′ 30″	134° 45′ 20″	21° 05′ 30″
Reversed (1)	173 02 00	134 45 10	20 53 25

Required the azimuth of the flag.

Latitude 48° 26′ 33″ N. Longitude 8 14^m W. (2)

Esquimalt M. T. Oct, 2,	3^h 20^m
Longitude	8 14 W.
Greenwich M. T. Oct. 2,	11^h 34^m
or	11^h, 6

Sun's declination on Oct. 2, at 0^h M. T. Gr.	3° 43′ 14″ S
Correction for 11^h 34^m	11 15
Sun's declination on Oct. 2, at 3^h 20^m M.T. Esqu.	3 54 29 S
Polar distance	93° 54′ 29″

Var. in 1^h	58″.2
Multiplied by	11 .6
	349.2
	582
	582
V. in 11^h 34^m=	675″.12
or	11′ 15″

Mean of the apparent altitudes	20° 59′ 27″
Refraction	2 31
Difference	20 56 56
Parallax	8
True altitude	20 57 04

(1) See note of Art. 30.

(2) In this example, the logarithms are taken out to the nearest 15″ only.

True altitude	20° 56′ 04″	colog. cosine	0.02970
Latitude	48 26 33	colog. cosine	0.17824
Polar distance	93 54 29		
Sum	163 18 06		
Half sum	81 39 03	log. cosine	9.16203
Half sum minus polar distance	12 15 26	log. cosine	9.98998

Sum	19.35995
Half sum or log. cosine 180° minus half the azimuth	9.67997
180° minus half the azimuth	61° 24′ 22″.5
Half the azimuth	118 35 37 .5
Azimuth of the sun	237 11 15
Mean of the H. C. R. on sun	172 49 45
Difference	64 21 30
Mean of the H. C. R. on flag.	134 45 15
Azimuth of the flag.	199° 06′ 45″

37. Azimuth by altitudes of stars.

The method is exactly the same as for the sun, and the same rules apply. The stars should be chosen as much East or West as possible, because they will, then, give more reliable results. For the same reason, the altitude should be small, but, on account of the uncertainty of refraction for small altitudes, it should not be less than 5°.

38. Graphic solution of the two preceding problems.

Describe a circle (Fig. 8 and 9) and draw two diameters, NS and QQ′, forming an angle equal to the colati-

tude. Make ACS equal to the altitude and QCB, to the sun's or star's declination, B being to the left of Q for Southern declinations and to the right for Northern ones. From A, let fall a perpendicular A*a* on NS, and from the centre C with the radius C*a*, describe a circle. Through A and B, draw A*s* and B*s* parallel to NS and QQ′. At *s*,

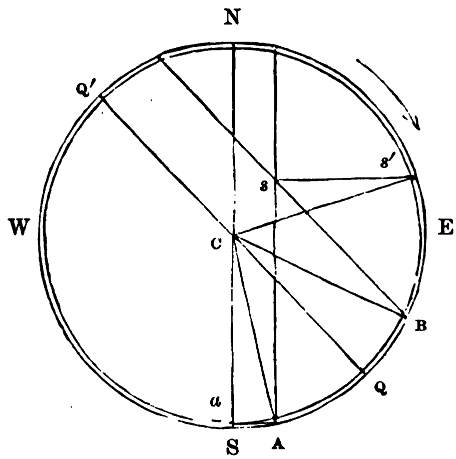


Fig. 8.

erect *ss*′ perpendicular to A*s*, on the left side if the star is West and on the right if East of the meridian. Join C to the point *s*′, where the perpendicular meets the inner circle, and the angle NC*s*′, reckoned in the direction of the arrow, is the azimuth. (1)

Fig. 8 is the solution of Example 1, Art. 36. SCQ is the colatitude of Pt. Fortune 44° 27′, SCA the true alti-

(1) See note of Art. 25.

tude 12° 42′, BCQ the declination North 21° 39′ and NCs' the sun's azimuth, 71° 55′.

Fig. 9 is the solution of Example 2, Art. 36. SCQ is

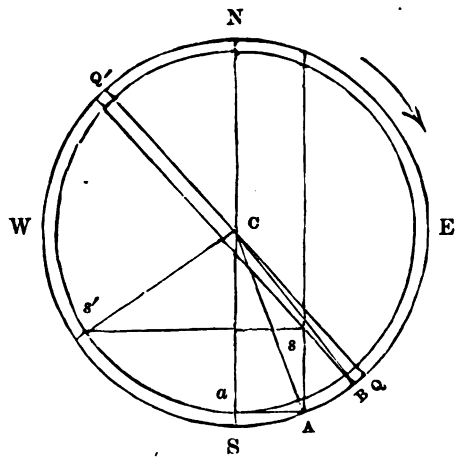


Fig. 9.

the colatitude of Esquimalt 41° 33′, SCA the true altitude 20° 57′, BCQ the declination South 3° 54′ and NCs' the sun's azimuth 237° 11′.

39. To find the variation of the compass.

When you observe, take the magnetic azimuth of the sun or star; the variation is the difference between the calculated and magnetic azimuths. It is East, when the calculated is greater than the magnetic and West, when it is less. If this difference was greater than 180°, 360° should be added to the smallest azimuth before subtracting.

Example 1.—The sun was observed to the N. 28° 20′ W. of the compass, its calculated azimuth was 325° 43′, required the variation.

Magnetic azimuth	331° 40′
Astronomical "	325 43
Variation	5° 57′ W.

Example 2.—A star was observed to the N. 8° 15′ E. of the compass, the calculated azimuth was 355° 43, required the variation.

Magnetic azimuth, 8° 15′ or	368° 15′
Astronomical "	355 43
Variation	12° 32′ W.

Example 3.—A star was observed to the N. 10° 30′ W. of the compass, the calculated azimuth was 7° 16′, required the variation.

Astronomical azimuth, 7° 16′ or	367° 16′
Magnetic "	349 30
Variation	17° 46′ E.

The magnetic needle and variation must be used only in compass surveying; in angular surveying they must be absolutely discarded. We know that the magnetic needle is not motionless; indeed, it may be said to be always moving. It is affected by diurnal, annual, secular and abnormal changes. At Toronto the difference caused by the former between 8 A.M. and 2 P.M. amounts, during some of the summer months, to 16′, and the abnormal changes have sometimes been greater than 2 degrees in 8 hours. (1) These numbers would be still

(1) See "Abstracts of Magnetical observations made at the Magnetical observatory, Toronto."

greater at places nearer the North magnetic pole, and it must not be forgotten that this pole is in Canada. (1) Moreover, considerable errors arise from the magnetic iron ores, so widely distributed among the lower geological strata which compose the Eastern part of the Dominion.

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(1) According to Gauss, it is in Latitude 73° 35′ N. and Longitude 95° 39′ W.

CHAPTER V.

CONVERGENCE OF MERIDIANS.

40. Definitions.

Let us consider two points, A and B, (Fig. 10,) situated in the Northern Hemisphere, and a third one, C, on the line AB (1), AN and CN′ being the meridians of A and C. The angle NAB is not equal to N′CB, it is less, and the difference is called the " convergence of meridians." But NAB is the azimuth of AB (Art. 29), and so is N′CB. We have therefore, two different azimuths for the line AB, or, rather, an infinity, as we should find a different one at each point D, E, F...... of AB. Consequently, it is not sufficient to give the azimuth of a line to determine its direction, the meridian from which it is reckoned, or, wich is the same thing, to which it is referred, must also be known.

In Fig. 10, NAB is the azimuih of AB reckoned from the meridian of A, and N′CB, the same azimuth reckoned

(1) This line is what is commonly understood as a straight line on the surface of the earth, namely, a geodetic line It is, here, assumed to be an arc of a great circle.

from the meridian of C. By substracting or adding the convergence to one of them, we obtain the other, that is, we refer it to the meridian of the other point.

Suppose that, having surveyed a line AGHK......, we deduce the azimuths of the lines AG, GH, HK,...... from the angles of the survey and the astronomical obser-

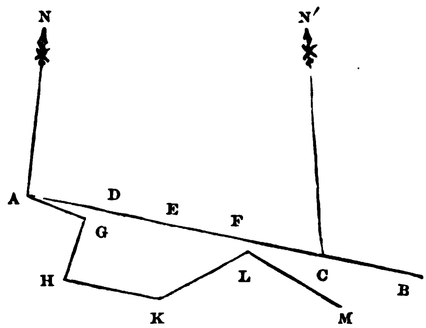

Fig. 10.

vations taken at A; any of those azimuths will then be reckoned from the meridian of A, that is to say, will be the same as would be found by direct observation on that meridian, if these lines were prolonged till they intersected the meridian.

41. To find the convergence of meridians between two points.

Find, by a traverse table or by logarithms the departure in Gunter's chains, add its logarithm to that of Table VI, col. 11, corresponding to the mean of the latitudes of the points. The sum is the logarithm of the convergence in seconds of arc.

Example.—The departure between two points is 473 chs., the mean latitude 48°, required the convergence.

Log. conver. for 1 ch. depart. (Table VI, col. 11)	9.85810
Log. 473	2.67486
Log. convergence	2.53296
Convergence	$341'' = 5'\ 41''$

42. To refer to the meridian of a point B an azimuth reckoned from the meridian of another point A.

If B is $\left\{ \begin{array}{c} \text{West} \\ \text{East} \end{array} \right\}$ of A, $\left\{ \begin{array}{c} \text{subtract} \\ \text{add} \end{array} \right\}$ the convergence between the points to the azimuth given.

Example.—Being in the Church steeple of Caughnawaga, near Montreal, the azimuth of a signal at Pte. Claire was observed and found equal to 276° 56′, the distance between the points was 581,55 chs. and the mean latitude 45° 26′. What would be the azimuth of the steeple if observed at the Pte. Claire signal ? (1)

By the traverse table, we have Pte. Claire 577.30 chs. W. of Caughnawaga.

Log. convergence for 1 ch. departure (Table VI, col. 11.)	9.82116
Log. 577.30	2.76140
Log. convergence	2.58256
Convergence	$382'' = 6'\ 22''$
Azimuth given + 180°	96° 56′ 00″
Azimuth at Pte. Claire	96° 49′ 38″

(1) **The azimuth given, being deduced from astronomical observation at Caughnawaga, is reckoned from the meridian of that place, and it is easily seen that the problem requires, in referring it to the meridian of Pte. Claire, the correction for convergence of meridians, besides adding 180°.**

43. To find the differences of latitude and longitude between two points, when their distance and azimuth are known.

Refer the azimuth to the meridians of both points and take the mean. Find, from it and the distance, the distance in latitude and departure. To the logarithm of the former, add that of Table VI. col, 2, corresponding to the mean latitude; the sum is the logarithm of the difference of latitude in seconds of arc.

Add together the log. of Table VI, col. 5, corresponding to the mean latitude and the logarithm of the departure; the sum is the logarithm of the difference of longitude in seconds of arc.

The mean latitude is necessary to take out the logarithms from Table VI. If that of one point only was known, the other would be found by an approximate calculation of the difference of latitude, using the given azimuth and latitude by the former method. This may be done by construction.

Example.—The azimuth of the Mission Church at Pte. Bleue, Lake St. John, P. Q., observed from a station on the East shore of the lake, is 298° 28′ and the distance 1395 chs. The East shore station is in latitude 42° 28′ 18″ N. and longitude 71° 58′ 31″ W. Required the latitude and longitude of the Church.

Mean latitude.

With 1395 chs. and 298° 28′ as azimuth, the traverse table gives:

Distance in latitude	665 chs. N.
Departure	1226 " W.

Log. 665	2.823
Log. 1 ch. in seconds of latitude (Table VI. col. 2).	9.814
Log. approximate difference of latitude	2.637
Approximate difference of latitude	434″=7′ 14″
Latitude of the East shore station	48° 28′ 18″
Approximate latitude of Church	48 35 32
Mean latitude	48° 31′ 55″

Mean azimuth.

Log. 1226	3.08849
Log. converg. for 1 ch. departure (Tab. VI, col. 11).	9.86627
Log. convergence	2.95476
Convergence	901″=15′ 01″
Azim. reckoned from East shore meridian	298° 28′ 00″
Azim. referred to Mission church meridian	298 12 59
Mean azimuth	298° 20′ 29″.5

Latitude.

Log. 1395	3.14457
Log. cosine 298° 20′ 29″	9.67614
Log. 1 ch. in seconds of latitude (Table VI, col. 2)	9.81380
Log. difference of latitude in seconds	22.63481
Difference of latitude	431″.3=7′ 11″.3 N
Latitude of East shore station	48° 28′ 18″.0
Latitude of Mission church	48° 35′ 29″.3

Longitude.

Log. 1395	3.14457
Log. sine 298° 20′ 29″.5	9.94455
Log. 1 ch. in seconds of longitude (Table VI, col. 5)	9.99155
Log. difference of longitude in seconds	23.08067
Difference of longitude	1204″=20′ 04″ W
Longitude of East shore station	71° 58′ 31″ W
Longitude of Mission church	72° 18′ 35″

44. Given the azimuths of a line at both ends, and the mean latitude, to find the length of the line.

Take the difference of the azimuths, it is the convergence. Reduce it to seconds, take its logarithm, add the cologarithm of convergence for 1 ch. departure, corresponding to the mean latitude, deduced from Table VI, col. 11, and the cologarithm sine of the mean azimuth The sum is the logarithm of the distance in chains. (1)

This method gives only an approximate result, and is of no use when the azimuths are near 0° or 180°. The azimuths ought to be very accurate and the points far apart.

Example.—From the top of Montreal mountain, the azimuth of Rigaud mountain was observed and found equal to 263° 53′ 33″ From the latter, the azimuth of the former was observed and found 83° 22′ 26″. Required the distance. Mean latitude 45° 29′.

(1) The formula is $D = \dfrac{Z' - Z}{c \sin\left(\dfrac{Z' + Z}{2}\right)}$, where Z and Z′ are the azimuths, c the convergence for 1 ch. (T. VI, col. 11) and D the distance in Gunter's chains.

1st Azimuth	263° 53′ 33″
2d "	263 22 26
Convergence	0 31 07 = 1867″
Mean azimuth	263 38 00

Log. 1867	3.27114
Col. convergence for 1 ch. departure (Table VI, col. 11)	0.18009
Col. sine mean azimuth	0.00269
Log. distance	3.45392
Distance	2844 chs.
	or 35 miles 44 chs.

45. Geodetic surveying.

Before going into the application of geodetic formulae, we shall make a brief exposition of the methods used in the angular survey of a line. There are three principal ones: by the angle of intersection, by the angle of deflection, and traversing (1).

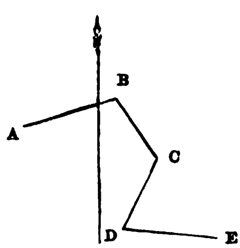

Fig. 11.

1*st Method.*—The angle of intersection is formed by two adjacent sides of the survey. Suppose we have to survey the line ABCDE (Fig. 11). Having adjusted the instrument at B, set the vernier at zero, and unclamp the horizontal circle. Direct the telescope to A, fasten the horizontal circle, and having loosened the vernier plate, sight to C by revolving the telescope

(1) Gillespie's Land Surveying, § 372 and 373; Gillespie's Higher Surveying § 94.

round the vertical axis; the H. C. R. is the angle of intersection. The same operation is then repeated at C and D. It is to be observed that, with an instrument graduated as in Fig. 5, the angles are reckoned from the station behind, through the right, to the station forward; they can, therefore, assume all the values between 0° and 360°.

Let the astronomical courses be:

Side.	Course.
A B	N. 76° E.
B C	S. 32 E.
C D	S. 26 W.
D E	S. 84 E.

Then the angles read on the circle and recorded in the field book, will be:

Station.	H. C. R.
B	252°
C	238
D	70

2d Method.—The angle of deflection is formed by one side produced and the next one. It is reckoned either from 0° to 360° or only from 0° to 180°, adding then the indication of right or left. To measure it, we proceed as in the first method, the only difference being that the telescope is turned over after the station behind has been sighted, and before directing it to the station forward. When the angles are reckoned from 0° to 360°, the angle of deflection is the reading of the circle; the field book would then show for the survey of ABCDE:

Station.	H. C. R.
B	72°
C	58
D	250

Otherwise, the angle being reckcned only from 0° to 180°, the designation "right" is given to the reading when it is less than 180° ; when it is more, the complement to 360° is taken, adding to it the designation "left." The survey of ABCDE, recorded by this method, would be :

Station.	Angle.
B	72° Right.
C	58 Right.
D	110 Left.

3d Method.—A traverse angle is formed by one of the sides and a parallel to a certain line adopted as the meridian of the survey. This line is not necessarily an astronomical or a magnetic meridian. To proceed by this method, set the instrument at B, fix the vernier at the assumed azimuth of AB, and unclamp the horizontal circle. Direct the telescope to A, fasten the horizontal circle, and, having loosened the vernier plate and turned over the telescope, sight to C. The H. C. R. is the traverse angle. The same operation is, then, repeated at C, D..., using at each station, the last H. C. R. recorded, in the same manner and instead of the assumed azimuth of AB.

If we take 76° for the azimuth of AB, the meridian of the survey is the astronomical meridian. At B, we set the vernier at 76° and find 148° for the H. C. R. At C,

we set the vernier at 148° and read 206°; at D, we set the vernier at 206° and read 96°. The record of the survey is, then:

Station.	H. C. R.
A	76°
B	148
C	206
D	96

A great improvement consists in measuring each angle twice with telescope in both positions, direct and reversed, using meridians differing by 180°. Accidental errors are thus easily discovered, and those caused by collimation and inclination of the horizontal axis are eliminated when the mean of the H. C. R. is used (the second one being diminished by 180° before taking the mean).

The following table, for instance would be the record of an accurate survey of ABCDE:

Station.	1st H. C. R.	2d H. C. R.	Mean or corrected H. C. R.
A	76° 00′	256° 00′	76° 00′
B	148 59	328 01	148 00
C	206 02	25 58	206 00
D	96 01	275 59	96 00

Of the three methods, traversing is the most accurate for making the plan. If we plot by latitudes and departures, whatever is the method applied in the field, we have to deduce the traverse angles from the record of the survey, in order to make use of the tables; therefore, we plot the survey as a traverse. Suppose now, we plot

with a protractor, and let any number, 5′ for instance, represent the possible error in drawing a line. The direction of a traverse course, on the plan, depends only on two lines: a parallel to the meridian and the course itself; the possible error, then, is only twice 5′. By other methods, it is as many times 5′ as there are courses before the one considered; and, in that case, the best way to make a correct plan would be to compute the angles which should have been observed in traversing, and plot the survey as a traverse.

Traversing has other advantages, which it would be too long to enumerate; we shall, therefore, in the following articles, suppose the surveys made by that method, the astronomical meridian of one of the stations being the meridian of the survey and the instrument so placed, that the horizontal circle readings at that station be 0° to North, 90° to East, 180° to South and 270° to West. The H. C. R. at any station will, thus, be an azimuth reckoned from the meridian of the survey. The station, at this meridian, we shall call astronomical (1).

When the survey is extensive, the orientation of the instrument must be rectified from time to time. The sun or a star will be observed as explained in chapter IV and the H. C. R. on sun noted at this second astronomical station. The difference between it and the calculated azimuth should be equal to the convergence of meridians between the astronomical stations, because the H. C. R. is the sun's azimuth, reckoned from the meridian of the first astronomical station, whilst the calculated azimuth is reckoned from the second. The error of orientation will consequently be found by referring one of the azimuths, the H. C. R. for instance, to the meridian of

(1) Because it is where the azimuth was observed.

the other The difference between them, if any, is the error. It is equally distributed among the courses by dividing it by the number of stations. Multiplying the result by the number of any course gives the correction for it.

Example.—The following traverse was made, the astronomical station being the first one:

Stations.	H. C. R.	Distances.
1	309° 39′ 15″	378.0 chs.
2	317 26 15	489.5
3	324 55 30	1154.6
4	290 37 15	702.2
5	272 17 30	642.0
6	256 31 45	588.4
7	259 42 15	846.0

At station 8, the sun was observed and its azimuth found by calculation 267° 11′ 50″, the H. C. R. on sun being 267° 59′ 10″. Required the error of orientation and the corrected traverse. Mean latitude 48° 39′. N.

Departure between St. 1 and St. 8,	3988.9 chs. W.
Log. 3988.9	3.60085
Log. convergence for 1 ch. departure (Table VI, col 11)	9.86830
Log. convergence	3.46915
Convergence	2945″ = 49′ 05″
H. C. R. on sun	267° 59′ 10″
H. C. R. on sun, referred to the meridian of St. 8	267 10 05
Azimuth of the sun at St. 8	267 11 50
Error of orientation	1′ 45″

Corrected traverse.

Stations.	H. C. R. observed.	Correction.	H. C. R. corrected.
1	309° 39′ 15″	+ 15″	309° 39′ 30″
2	317 26 15	30	317 26 45
3	324 55 30	45	324 56 15
4	290 37 15	1′ 00	290 38 15
5	272 17 30	1 15	272 18 45
6	256 31 45	1 30	256 33 15
7	259 42 15	1 45	259 44 00

46. Plotting the survey.

From the last station, the survey might be carried on with the same meridian as before, and plotted with it, the only correction to the course being the error of orientation; but it is more simple to use the meridian of the last astronomical station and lay it down on the plan, making with the first one an angle equal to the convergence between them. Each course, then, is plotted with the meridian it is reckoned from.

According to it, in the example of Art. 45, the last course 259° 42′ 15″, should be corrected by the difference between the calculated azimuth and the H. C. R. on sun, 47′ 20″. At St. 8, the instrument should be set up, and the survey carried onward with that corrected course, 258° 54′ 55″. Courses from 1 to 8 should be plotted as is usually done, and at St. 8, a meridian drawn making with the first one an angle equal to the convergence, 49′ 05″, the following courses being plotted with it as a meridian.

It is not necessary to correct the orientation immediately after the observation is taken; the survey may be continued, and the correction applied only when the

day's work is over, or even when the survey is entirely completed.

47. To run a line.

We know (Art. 40) that the azimuth of a line is not the same at any of its points, except when its direction is North and South. Its angle with the meridian, at any station, will be found by referring the initial azimuth to that point.

Example.—Starting from a place in latitude 46° 05′ N, a surveyor runs an exploratory line on the azimuth 285°. At the 60th mile, he takes astronomical observations to rectify its course; on what azimuth must he produce the line as a straight (1) line ? (2)

Mean latitude.

Distance in latitude for the azimuth 285° and the distance 4800 chs	1242 chs. N.
Log. 1242	3.094
Log. 1 ch. in seconds of latitude (Table VI, col. 2)	9.814
Log. difference of latitude in seconds	12.908
Difference of latitude	809″=13′ 29″
Latitude of initial point	46° 04′ 00″
Latitude of 60th mile	46 17 29
Mean latitude	46° 11′

(1) A geodetic line.

(2) In this example, an approximate difference of latitude is found as in example of Art. 43, and that of the 60th mile and mean latitude deduced. With the latter, the initial azimuth is referred to the meridian of the 60th mile.

Azimuth.

Departure for the azimuth 285° and the distance 4800 chs	4636.5 chs E
Log. 4636.5	3.66619
Log. convergence for 1 ch. departure (Table VI, col. 11)	9.83053
Log. convergence	13.49672
Convergence	3138″=52′ 18″
Initial azimuth	285° 00′ 00″
Azimuth at the 60th mile	284° 07′ 42″

48. To lay out on the ground, a figure of which the plan is given, correcting the course by astronomical observations.

Each side must be run with the azimuth reckoned from the meridian where the course was corrected. Suppose, for instance, the figure to be a square township and the course to be corrected by observations at the four corners. The azimuth of each side, deduced from the plan, must be referred to the meridian of the corner which is its initial point.

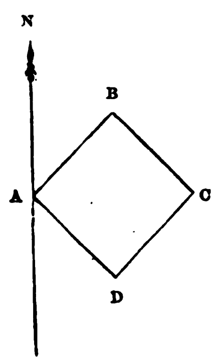

Fig. 12.

Example.—Required the azimuths to be used in running the exterior lines of a square township, ABCD, (Fig. 12) whose sides are 8 miles and initial azimuth NAB, 45°, correcting the course by observations at the four corners. Mean latitude 47° N.

Azimuts reckoned from the meridian of A	BC	CD	DA
	135°	225°	315°

Departure between	A and B	A and C	B and C
	452.55	905.10	452.55 chs.
Log. departure	2.65567	2.95670	
Log. convergence for 1 ch. departure	9.84290	9.84290	
Log. convergence	2.49857	2.79960	
Convergence	315″=5′ 15″	630″=10′ 30″	5′ 15″
Azimuth reckoned from the meridian of A	135° 00′ 00″	225° 00′ 00″	315° 00′ 00″
Azim. to be used	135° 05′ 15″	225° 10′ 30″	315° 05′ 15″

49. Given the azimuths of sides, to find the angles of a triangle.

Sometimes the angles of a triangle cannot be measured directly, either because one of the angular points of the triangle cannot be seen from another or for any other reason. But they may be found if the azimuths of the sides have been observed, by referring them to the same meridian and taking the difference.

Example.—From a station on the Eastern Shore of Lake St. John, the observed azimuth of the Mission Church at Pte. Bleue was 298° 28′; a traverse between that station and the mouth of the Peribonka river, being worked out gave for the distance 1827 chs. and for azimuth 323° 28′. At the mouth of the Peribonka river, the observed azimuth of the Mission Church was 189° 35′. Required the angles of the triangle East Shore, Peribonka, Mission Church. Mean latitude 48° 37′ N. (1)

(1) In this example, the azimuths of the traverse and of the Church from the East Shore Station, being reckoned from the meridian of that station, the third azimuth is referred to it and the differences taken.

Departure for the azimuth 323° 28′ and the distance 1827 chs.	1087.5 chs.
Log. 1087.5	3.03643
Log. convergence for 1 ch. (Table VI, col. 11)	9.86754
Log. convergence	2.90397
Convergence	802″=13′ 22″
Azimuth reckoned from the meridian of Peribonka	189° 35′ 00″
Azimuth referred to the meridian of E. Shore Station	189° 48′ 22″

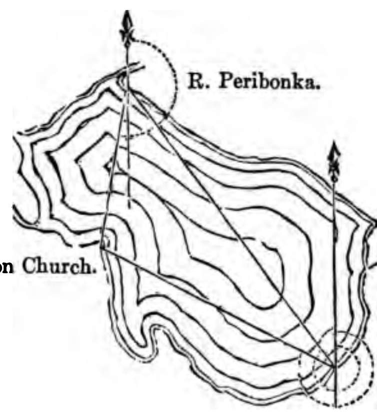

Fig. 13.

Angle at Peribonka.

Azimuth of Church	189° 48′ 22″
" East Shore	143 28 00
Angle at Peribonka	46° 20′ 22″

Angle at Mission Church.

Azimuth of East Shore	118° 28′ 00″
" Peribonka	9 48 22
Angle at Mission Church	108° 39′ 38″

Angle at East Shore.

Azimuth of Mission Church	323° 28′ 00″
" Peribonka	298 28 00
Angle at East Shore	25° 00′ 00″

50. To produce a parallel of latitude by laying out chords of a given length.

The angle of deflection between two chords, BAD, Fig. 14, is equal to the convergence of meridians for the

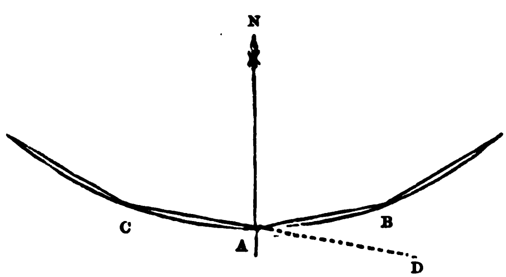

Fig. 14.

length of a chord, and the azimuths NAB and NAC, are equal to 90° minus half the convergence and 270° plus the same.

Example.—To lay out, by chords 489 chs. long the parallel of 49° 21′ N.

Log. 489)	2.68931
Log. convergence for 1 ch. departure (Table VI, col. 11).	9.87871
Log. convergence	12.56802
Convergence, or deflection angle	370″=6′ 10″
½ Convergence	3′ 05″
Azimuths of chords	89° 56′ 55″—270° 03′ 05″

This parallel is the second base line in Manitoba and the North West Territory. The chords are the bases of the townships, the East and West sides being meridians. Since they converge, the North base is smaller than the South one. The difference is equal to the length of side, 489 chs, multiplied by the sine of the convergence. This angle being small, its sine is equal to the number of seconds multiplied by sine 1″.

Log. 489	2.68931
Log. convergence	2.56802
Log. sine 1″	4.68557
Log. difference	9.94290
Difference	0 ch. 88 links.

51. To lay out a parallel of latitude by offsets.

A parallel may be laid out by running a line perpendicular to the meridian and measuring offsets to the North. The length of an offset is equal to the square of its distance from the meridian multiplied by the sine of half the convergence of meridians for 1 ch. departure. This angle being small its logarithm sine will be obtained

by adding logarithm sine ½ second to the logarithm of the convergence for 1 ch. departure (Table VI. col. 11). When the offsets are equidistant, any of them is obtained by multiplying the first one by the square of the number representing the offset.

Example.—To lay out the 49th parallel of latitude by means of offsets at each mile (1).

2 × log. 80		3.80618	
Log. convergence for 1 ch. departure		9.87335	
Log. sine ½ second		4.38454	
Log. 1st. offset		18.06407	
1st. offset	=	0. ch., 01159 =	1 links
2d "	=	0. 1159 × 4 =	5 "
3d "	=	0. 1159 × 9 =	10 "
4th "	=	0. 1159 ×16 =	18 "
5th "	=	0. 1159 ×25 =	29 "
6th "	=	0. 1159 ×36 =	42 "

52. Projections.

The preceding method may be used for making projections. The offsets may be calculated for each of the parallels of the map, adopting, for their interval, a number of miles suited to the scale. Having drawn two perpendicular lines, NS and WE, (Fig. 15), these offsets will be platted at 1, 2, ... and the points obtained joined by straight lines. In table VI, col 7, the distance

(1) This parallel is part of the boundary between Canada and the United States. It was laid out, by the International Boundary Commission, as described here.

in chains, between the parallel WE and the next one, will be taken out, set off from C to D and FG plotted as

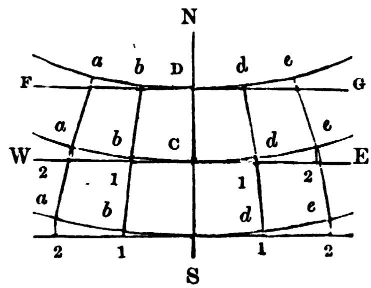

Fig. 15.

EW. All the parallels being projected, the distance on each of them, between two meridians will be taken out of Table VI col. 9, set off on the map, and the points *a, a, a,*......, *b, b, b,*......, *d, d, d,*...... joined by straight lines. This projection is about the same as the polyconic, when the map does not extend more than 30 or 40 degrees. For larger maps, special tables should be used (1).

Supposing the meridians and parallels to be marked for each degree, C being in Latitude 45°, we find for the distance between the meridians on the 45th parallel 3919 chs. (Table VI, col. 9) and for the corresponding offsets:

1	2	3	4
24.2	96.7	217.6	386.9 chs.

CD, distance between the 45th and 46th parallels is equal to 5524, 3 chs. (Table VI, col. 7, for 45° 30′ latitude).

(1) Lee's tables—Governement Printing Office, Washington, may be recommended.

On the 46th parallel, we find in the same manner:

Distance between the meridians 3850, 2 chs.

	1	2	3	4
Offsets	24.2	96.7	217.5	386.7 chs.

and so on for other parallels.

CHAPTER VI.

SUN DIALS.

53. Horizontal dial.

Fix on a plate NS (fig. 16) a triangular sheet of metal BAC, whose angle CBA is equal to the latitude, the plane of the triangle being perpendicular to the plate. This last condition is attained by presenting a square on

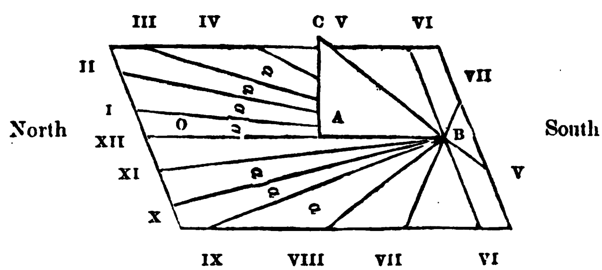

Fig. 16.

both sides of ABC. With a spirit level, make the plate BO horizontal, the line BA being nearly in the meridian and the hypothenuse BC facing to South. Observe the sun's meridian transit and turn the plate BO, keeping it horizontal, till the shadow of the triangle is exactly on the prolongation AO of BA, when the sun is on the me-

ridian. Fix the dial firmly in that position and, with a good watch set to noon when the shadow is on AO, mark the points *a*, *a*, *a*, *a*...... where the shadow is at 1, 2, 3..... P. M., 5, 6, 7...... A. M. Join B*a*, B*a*, B*a*, which are the hour lines.

These lines may be drawn directly as follows. Let BO, fig. 17, be the dial's meridian or noon line. Draw BC, making with BO an angle OBC equal to the latitude. At any point C of BC, erect a perpendicular CE, measure

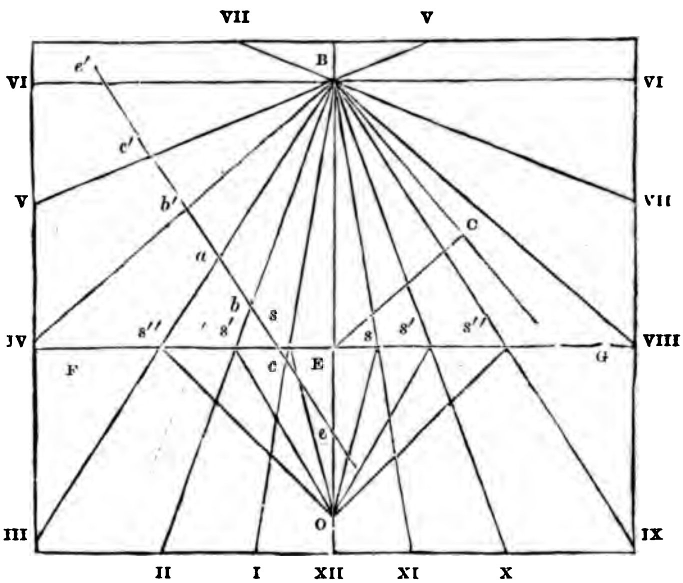

Fig. 17.

EO equal to EC, and at E, erect the perpendicular FG to OB. Through O, draw the lines *os*, *os'*, *os''*...... inclined of 15°, 30°, 45°...... on OB. Join B*s*, B*s'*, B*s''*...... which are the hour lines. For some of them, the construction would extend outside the plate, but they may be found

by drawing parallels to the 9 A. M. and 3 P. M. lines, measuring *ab′* equal to *ab*, *ac′* to *ac*, *ae′* to *ae*, and joining B*b′*, B*c′*, B*e′*.

We supposed that the sun was observed with a transit. When none is at hand, choose a level surface or piece of ground, and plant on it a staff vertically by means of a plumb line. Describe several circles from the foot of the staff as a centre, and note the time of a clock or watch when the extremity of the shadow crosses each of them. The mean (1) will be the time shewn by the clock or watch at the sun's meridian transit, and the dial will be in the right position if the shadow is on the line BO at that time.

A more satisfactory method by means of Alioth or γ Cassiopeiae and Polaris, is given at Art. 31. To apply it, suspend a plumb line to a tree or an elevated point

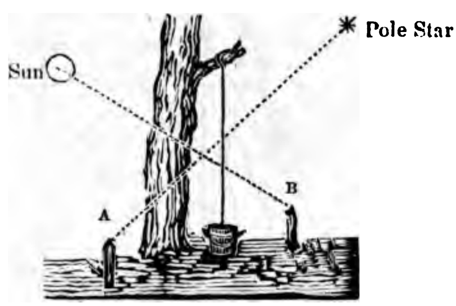

Fig. 18.

and let the bob dip in a pail of water, to lessen the oscillations. Place a light behind you, to illuminate the plumb line and render it visible. The time when both

(1) Noticing that the mean of A. M. and P. M. times, such as 9 A. M. and 1 P. M. is 11 A. M. and not 5 o'clock.

stars are covered by it is noted, and 24 or 26 minutes after when Polaris is on the meridian, a stake A is placed so that, looking from its top, the star is covered by the plumb line. Another stake will, then, be planted at B, so that, looking from it, the other A is covered by the plumb line whose shadow, cast by the sun, will fall on B, at the time of meridian transit or apparent noon.

54. Vertical dial.

This dial is formed by a style, fixed on the side of a wall, and bearing at the end a disc nearly perpendicular to the sun's rays at noon, in March or September. At its centre, a hole with sharp edges, projects a bright spot on the wall, in the middle of the disc's shadow, cast by the

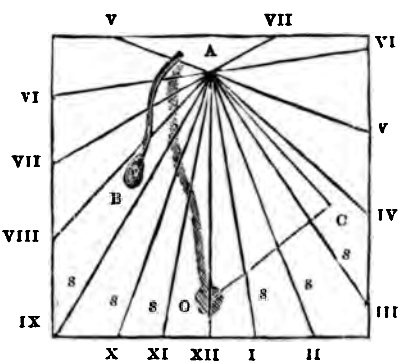

Fig. 19.

sun. The position of that spot among the hours lines indicates the time. This dial has some advantages; it can be constructed of large dimensions and placed on a high wall, so as to be seen from a great distance. It is, also, more easily made accurate.

To graduate it, proceed as for the other dial, at the sun's meridian transit, and mark the place O (Fig. 19) at that time, of the bright spot. By means of a plumb line, draw a vertical line through that point; it is the noon line. Measure the distance between O and the middle of the hole B, and describe the triangle AOC, where CO is equal to the measured distance OB, AOC to the sun's polar distance minus the colatitude and OCA to the supplement of the polar distance. With a good watch, set to noon at the time of the sun's transit, mark for each successive hour, the positions *s*, *s*, *s*,...... of the bright spot on the wall and join A*s*, A*s*, A*s*,...... which are the hour lines.

There are several methods of drawing those lines directly; but, although not very complicated, they would be out of place in this work.

Sun dials show the apparent time, and we know (Art. 4) that civil is mean time. The clocks, therefore, must not be set to the time shewn by the dial, it must be previously corrected by Table VII.

TABLE I.

MEAN REFRACTION. (1)

Apparent altitude.	Refraction.	Apparent altitude.	Refraction.	Apparent altitude.	Refraction.	Apparent altitude.	Refraction.	Apparent altitude.	Refraction.
° ′	′ ″	° ′	′ ″	° ′	′ ″	°	′ ″	°	′ ″
0 00	36 00	14 00	3 50	29 30	1 43	45	0 58	76	0 15
20	31 00	30	3 42	30 00	1 41	46	0 56	77	0 13
40	28 00	15 00	3 34	30	1 39	47	0 54	78	0 12
1 00	25 00	30	3 27	31 00	1 37	48	0 52	79	0 11
20	22 00	16 00	3 21	30	1 35	49	0 51	80	0 10
40	20 00	30	3 14	32 00	1 33	50	0 49	81	0 09
2 00	18 26	17 00	3 08	30	1 31	51	0 47	82	0 08
20	16 55	30	3 03	33 00	1 30	52	0 46	83	0 07
40	15 35	18 00	2 58	30	1 28	53	0 44	84	0 06
3 00	14 25	30	2 52	34 00	1 26	54	0 42	85	0 05
30	12 57	19 00	2 48	30	1 25	55	0 41	86	0 04
4 00	11 44	30	2 43	35 00	1 23	56	0 39	87	0 03
30	10 44	20 00	2 39	30	1 22	57	0 38	88	0 02
5 00	9 52	30	2 35	36 00	1 20	58	0 36	89	0 01
30	9 07	21 00	2 31	30	1 19	59	0 35	90	0 00
6 00	8 28	30	2 27	37 00	1 17	60	0 34		
30	7 54	22 00	2 23	30	1 16	61	0 32		
7 00	7 24	30	2 20	38 00	1 15	62	0 31		
30	6 57	23 00	2 16	30	1 13	63	0 30		
8 00	6 33	30	2 13	39 00	1 12	64	0 28		
30	6 12	24 00	2 10	30	1 11	65	0 27		
9 00	5 53	30	2 07	40 00	1 09	66	0 26		
30	5 35	25 00	2 04	30	1 08	67	0 25		
10 00	5 19	30	2 02	41 00	1 07	68	0 24		
30	5 05	26 00	1 59	30	1 06	69	0 22		
11 00	4 51	30	1 56	42 00	1 05	70	0 21		
00	4 39	27 00	1 54	30	1 04	71	0 20		
12 00	4 28	30	1 52	43 00	1 02	72	0 19		
30	4 17	28 00	1 49	30	1 01	73	0 18		
13 00	4 07	30	1 47	44 00	1 00	74	0 17		
30	3 58	29 00	1 45	30	0 59	75	0 16		

TABLE II.
SUN'S
Paral in altitude.

Altit.	Parallax.
0	9
10	9
20	8
30	8
40	7
50	6
60	5
70	3
80	2
90	0

(1) Barometer 30 inches: Fahrenheit Thermometer + 50°.

TABLE III.

Times of elongation of the pole star, on the 1st, 11th and 21st of each month for latitude 47° N. Longitude 5^h W. and the year 1878. (1)

Month.	1st.			Elongation	11th.			Elongation	21st.			Elongation
January...	0^h	25^m	A.M.	W	11^h	42^m	P. M.	W	11^h	03^m	P. M.	W
February.	10	20	P.M.	"	9	40	"	"	9	01	"	"
March....	8	29	"	"	7	50	"	"	7	11	"	"
April	6	28	"	"	6	02	A.M.	E	5	22	A.M.	E
May.......	4	43	A.M.	E	4	04	"	"	3	24	"	"
June.	2	41	"	"	2	02	"	"	1	23	"	"
July.......	0	43	"	"	0	04	"	"	11	21	P. M.	"
August....	10	37	P.M.	"	9	58	P.M.	"	9	19	"	"
September	8	34	"	"	7	56	"	"	7	17	"	"
October...	6	38	"	"	5	58	"	"	5	09	A.M.	W
November	4	26	A.M.	W	3	47	A.M.	W	3	08	"	"
December	2	28	"	"	1	49	"	"	1	10	"	"

(1) These times increase by 21^s yearly, about.

TABLE IV.

Azimuth of the pole star at Eastern elongation (1) for the latitudes 42° to 54 North and the years 1878 to 1890.

Latitude.	1878.	1879.	1880.	1881.	1882.	1883.	1884.	1885.	1886.	1887.	1888.	1889.	1890.	Latitude.
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	°
42	1 48	1 48	1 47	1 47	1 47	1 46	1 46	1 45	1 45	1 44	1 44	1 44	1 43	42
43	1 50	1 50	1 49	1 49	1 48	1 48	1 47	1 47	1 47	1 46	1 46	1 45	1 45	43
44	1 52	1 51	1 51	1 51	1 50	1 50	1 49	1 49	1 48	1 48	1 47	1 47	1 47	44
45	1 54	1 53	1 53	1 53	1 52	1 52	1 51	1 51	1 50	1 50	1 49	1 49	1 48	45
46	1 56	1 55	1 55	1 55	1 54	1 54	1 53	1 53	1 52	1 52	1 51	1 51	1 50	46
47	1 58	1 58	1 57	1 57	1 56	1 56	1 55	1 55	1 54	1 54	1 53	1 53	1 52	47
48	2 00	2 00	1 59	1 59	1 58	1 58	1 57	1 57	1 57	1 56	1 56	1 55	1 55	48
49	2 03	2 02	2 02	2 01	2 01	2 00	2 00	1 59	1 59	1 58	1 58	1 57	1 57	49
50	2 05	2 05	2 04	2 04	2 03	2 03	2 02	2 02	2 01	2 01	2 00	2 00	1 59	50
51	2 08	2 07	2 07	2 06	2 06	2 05	2 05	2 04	2 04	2 03	2 03	2 02	2 02	51
52	2 11	2 10	2 10	2 09	2 09	2 08	2 08	2 07	2 07	2 06	2 06	2 05	2 05	52
53	2 14	2 13	2 13	2 12	2 12	2 11	2 11	2 10	2 10	2 09	2 08	2 08	2 07	53
54	2 17	2 16	2 16	2 15	2 15	2 14	2 14	2 13	2 13	2 12	2 12	2 11	2 10	54

(1) Subtract it from 360° to have the azimuth at Western elongation.

TABLE V.

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star / Magnitude. / S.T. when vertic. with Polaris. / Colog. sine distance from Polaris.	ε Ursae Majoris (Alioth) 2 / 0^h 50^m / 0.24456			η Ursae Majoris. 2 / 1^h 41^m / 0.17952			β Ursae Minoris. 2 / 2^h 41^m / 0.54464			γ Eridani. 3 / 3^h 48^m / 0 01096			
Latitude.	North altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	South altitude	Azimuth.	Annual increase	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	8 43	0 11.5	5	2 12	359 47 0	12	26 41	359 18.5	18	34 10	358 51.5	23	42
43	9 43	" 12.0	"	3 09	46.5	"	27 41	" 18.0	"	33 10	" 50.0	"	43
44	10 42	" 12.0	6	4 07	46.5	"	28 41	" 17.5	"	32 11	" 49.0	24	44
45	11 42	" 12.0	"	5 05	46.0	"	29 41	" 16.5	19	31 11	" 48.0	"	45
46	12 41	" 12.5	"	6 04	46.0	"	30 41	" 16.0	"	30 11	" 46 5	"	46
47	13 41	" 12.5	"	7 03	45.5	"	31 41	" 15.0	"	29 11	" 45.0	25	47
48	14 41	" 13.0	"	8 02	45.5	13	32 41	" 14.0	20	28 11	" 43.5	"	48
49	15 40	" 13.0	"	9 01	45.0	"	33 40	" 13.0	"	27 11	" 42.0	26	49
50	16 40	" 13.5	"	10 00	45.0	"	34 40	" 12.0	"	26 11	" 40.5	"	50
51	17 40	" 13.5	"	11 00	44.5	14	35 40	" 11.0	21	25 11	" 39.0	27	51
52	18 40	" 14.0	"	12 00	44.0	"	36 40	" 10.0	"	24 11	" 37.0	28	52
53	19 40	" 14.5	7	12 59	43.5	"	37 40	" 9.0	22	23 11	" 35.0	"	53
54	20 40	" 14.5	"	13 59	43.5	"	38 40	" 8.0	"	22 11	" 33.0	29	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star. Magnitude. S.T. when vertic. with Polaris Colog. sine distance from Polaris.	η Draconis. 3.2 $4^h\ 11^m$ 0.31256			β Orionis (Rigel). 1 $5^h\ 03^m$ 0.00388			β Draconis. 3.2 $5^h\ 17^m$ 0.20848			γ Draconis. 2.3 $5^h\ 42^m$ 0.20136			
Latitude.	North altitude	Azimuth.	Annual increase	South altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	13 51	358 42.5	24	39 40	358 27.5	26	4 34	358 24.5	27	3 43	358 19.5	27	42
43	14 51	" 42.0	25	38 40	26.0	"	5 33	" 23.0	"	4 41	" 17.5	"	43
44	15 50	" 40.5	"	37 40	24.5	"	6 32	" 21.5	28	5 39	" 16.0	28	44
45	16 50	" 39.0	"	36 40	23.0	27	7 31	" 19.5	"	6 38	" 14.0	"	45
46	17 50	" 37.5	26	35 40	21.0	"	8 30	" 18.0	29	7 37	" 12.5	29	46
47	18 50	" 36.0	"	34 40	19.5	28	9 30	" 16.0	"	8 36	" 10.5	"	47
48	19 50	" 34.5	27	33 40	17.5	"	10 29	" 14.0	30	9 36	" 8.0	30	48
49	20 50	" 33.0	"	32 41	15.5	29	11 29	" 12.0	"	10 35	" 6.0	31	49
50	21 49	" 31.0	28	31 41	13.5	30	12 28	" 9.5	31	11 35	" 3.5	"	50
51	22 49	" 29.0	29	30 41	11.0	"	13 28	" 7.0	32	12 34	" 1.0	32	51
52	23 49	" 27.0	"	29 41	8.5	31	14 28	" 4.5	"	13 34	357 58.5	33	52
53	24 49	" 25.0	30	28 41	6.0	32	15 27	" 2.0	33	14 34	" 55.5	"	53
54	25 49	" 22.5	31	27 41	3.0	33	16 27	357 59.0	34	15 33	" 52.5	34	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star	α Canis Majoris (Sirius).			δ Draconis.			α Canis Minoris (Procyon)			α Cephei.			
Magnitude.	1			3			1			3.2			
S.T. when vertic. with Polaris	$6^h\ 33^m$			$6^h\ 54^m$			$7^h\ 28^m$			$9^h\ 02^m$			
Colog sine distance from Polaris.	0.01792			0.41534			0.00195			0.33889			
Latitude.	South altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	South altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	31 29	358 13.0	27	19 30	358 12.0	26	53 33	358 12.0	26	14 08	358 24.5	18	42
43	30 29	" 11.0	"	20 30	" 10.0	"	52 33	" 10.0	"	15 08	23.0	"	43
44	29 29	" 9.0	28	21 29	" 8.5	"	51 33	" 8.5	"	16 07	21.5	"	44
45	28 29	" 7.5	"	22 29	" 6.5	27	50 33	" 6.5	27	17 07	19.5	19	45
46	27 29	" 5.5	29	23 29	" 4.5	"	49 33	" 4.5	"	18 07	18.0	"	46
47	26 29	" 3.0	"	24 29	" 2.0	28	48 33	" 2.0	28	19 07	16.0	"	47
48	25 29	" 1.0	30	25 29	" 0.0	"	47 33	" 0.0	"	20 07	14.0	20	48
49	24 29	357 58.5	"	26 29	357 57.5	29	46 33	357 57.5	29	21 07	12.0	"	49
50	23 29	" 56.5	31	27 29	" 55.0	30	45 33	" 55.0	30	22 06	9.5	21	50
51	22 29	" 53.5	32	28 29	" 52.5	"	44 33	" 52.5	"	23 06	7.0	"	51
52	21 29	" 50.5	"	29 29	" 49.5	31	43 33	" 49.5	31	24 06	4.5	22	52
53	20 30	" 47.5	33	30 29	" 46.5	32	42 33	" 46.5	32	25 06	2.0	"	53
54	19 30	" 44.5	34	31 29	" 43.5	"	41 33	" 43.5	"	26 06	357 59.0	"	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star Magnitude. S.T. when vertic. with Polaris Colog. sine distance from Polaris.	β Cephei. 3.2 $9^h\ 09^m$ 0.48144			α Hydrae. 2 $9^h\ 16^m$ 0.00518			γ Cephei. 3.4 $11^h\ 21^m$ 0.68765			α Cassiopeiae. 2.3 $12^h\ 31^m$ 0.26609			
Latitude.	North altitude	Azimuth.	Annual increase	South altitude	Azimuth.	Annual increase	North altitude	Azimuth.	Annual increase	North altitude	Azimuth.	An. decrease	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	22 03	358 26.0	17	39 53	358 28.0	17	28 59	359 10.0	4	7 59	359 40.5	4	42
43	23 03	" 24.5	"	38 53	" 26.5	"	29 59	" 9.0	"	8 58	" 40.0	"	43
44	24 03	" 23.0	"	37 53	" 25.0	"	30 59	" 8.0	"	9 57	" 39.5	"	44
45	25 03	" 21.5	"	36 53	" 23.0	"	31 59	" 7.0	"	10 57	" 39.5	"	45
46	26 03	" 19.5	18	35 53	" 21.5	18	32 59	" 6.5	"	11 57	" 39.0	"	46
47	27 03	" 18.0	"	34 53	" 19.5	"	33 58	" 5.5	"	12 56	" 38.5	"	47
48	28 03	" 16.0	19	33 53	" 17.5	19	34 58	" 4.0	"	13 56	" 38.0	"	48
49	29 03	" 14.0	"	32 54	" 15.5	"	35 58	" 3.0	5	14 56	" 37.5	5	49
50	30 03	" 11.5	"	31 54	" 13.5	"	36 58	" 2.0	"	15 55	" 37.0	"	50
51	31 03	" 9.5	20	30 54	" 11.0	20	37 58	" 0.5	"	16 55	" 36.5	"	51
52	32 02	" 7.0	"	29 54	" 9.0	"	38 58	358 59.5	"	17 55	" 36.0	"	52
53	33 02	" 4.0	"	28 54	" 6.5	"	39 58	" 58.5	"	18 55	" 35.5	"	53
54	34 02	" 1.5	21	27 54	" 3.5	21	40 58	" 56.5	"	19 55	" 35.0	"	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star / Magnitude. / S.T. when vertic with Polaris. / Colog. sine distance from Polaris.	γ Cassiopeiae. 2 / $12^h\ 48^m$ / 0.31980			α Virginis (Spica). 1 / $13^h\ 19^m$ / 0.00938			β Librae. 2 / $15^h\ 14^m$ / 0.00678			α Persei. 2 / $15^h\ 22^m$ / 0 19722			
Latitude.	North altitude	Azimuth.	An. decrease	South altitude	Azimuth.	An. decrease	South altitude	Azimuth.	An. decrease	North altitude	Azimuth.	An. decrease	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	12 07	359 48.0	5	37 30	0 02 5	8	39 05	0 53.0	20	1 45	0 56.0	20	42
43	13 07	" 47.5	"	36 30	" 2.5	9	38 05	" 53.5	21	2 41	" 57.0	21	43
44	14 07	" 47.5	6	35 30	" 2.5	"	37 05	" 54.5	"	3 38	" 58.0	"	44
45	15 07	" 47.0	"	34 30	" 2.5	"	36 05	" 55.5	"	4 36	" 59.0	"	45
46	16 06	" 47.0	"	33 30	" 2.5	"	35 05	" 56.5	22	5 35	1 00 0	22	46
47	17 06	" 47.0	"	32 31	" 2.5	"	34 05	" 57.5	"	6 34	" 01.0	"	47
48	18 06	" 46.5	6	31 31	" 2.5	10	33 05	" 58.5	"	7 33	" 2.5	"	48
49	19 06	" 46.5	"	30 31	" 2.5	"	32 06	1 00.0	23	8 32	" 3.5	23	49
50	20 06	" 46.0	"	29 31	" 2.5	"	31 06	" 1.0	"	9 32	" 5.0	"	50
51	21 06	" 45.5	"	28 31	" 2.5	"	30 06	" 2.5	24	10 31	" 6.5	24	51
52	22 05	" 45.5	7	27 31	" 2.5	"	29 06	" 4.0	"	11 31	" 7.5	"	52
53	23 05	" 45.0	"	26 31	" 3.0	11	28 06	" 5.5	25	12 30	" 9.0	25	53
54	24 05	" 44.5	"	25 31	" 3.0	"	27 06	" 7.0	26	13 30	" 11.0	26	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star Magnitude. S.T. when vertic. with Polaris Colog. sine distance from Polaris.	α Scorpii (Antares). 1.2 16^h 28^m 0.05039			θ Ophiuci. 3.4 17^h 21^m 0.04466			σ Sagittarii. 2.3 18^h 56^m 0.04856			α Capricorni. 3.4 20^h 18^m 0.01056			
Latitude.	South altitude	Azimuth.	An. decrease.	South altitude	Azimuth.	An. decrease.	South altitude	Azimuth.	An. decrease.	South altitude	Azimuth.	An. decrease.	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	21 53	1 20.0	24	23 09	1 34.5	27	21 35	1 47.5	26	35 06	1 45.0	23	42
43	20 53	" 21.0	25	22 09	" 36.0	"	20 36	" 49.5	"	34 06	" 46.5	"	43
44	19 53	" 22.5	"	21 09	" 37.5	28	19 36	" 51.5	"	33 06	" 48.5	24	44
45	18 53	" 24.0	26	20 10	" 39.0	"	18 36	" 53.0	27	32 07	" 50.0	"	45
46	17 53	" 25.5	"	19 10	" 41.0	29	17 36	" 55.0	"	31 07	" 52.0	"	46
47	16 54	" 27.0	27	18 10	" 43.0	"	16 36	" 57.5	28	30 07	" 54.0	25	47
48	15 54	" 28.5	"	17 10	" 45.0	30	15 36	" 59.5	"	29 07	" 56.5	"	48
49	14 54	" 30.5	28	16 10	" 47.0	"	14 37	2 02.0	29	28 07	" 58.5	26	49
50	13 54	" 32.5	"	15 11	" 49.0	31	13 37	" 4.5	30	27 07	2 01.0	"	50
51	12 55	" 34.5	29	14 11	" 51.5	32	12 37	" 7.0	"	26 07	" 04.0	27	51
52	11 55	" 36.5	"	13 11	" 54.0	"	11 38	" 10.0	31	25 07	" 6.5	28	52
53	10 55	" 38.5	30	12 11	" 56.5	33	10 38	" 13.0	32	24 07	" 9.5	"	53
54	9 55	" 41.0	31	11 12	" 59.5	34	9 39	" 16.0	"	23 07	" 12.5	29	54

TABLE V.—*Continued.*

For finding the Azimuth of Polaris when vertical with other stars, for the year 1878 and the latitudes 42° to 54° N.

Star Magnitude. S.T. when vertic with Polaris. Colog. sine distance from Polaris.	θ Ursae Majoris. 3 $21^h\ 35^m$ 0.20581			α Piscis Australis (Fomalhaut). 1.2 $22^h\ 56^m$ 0.05893			α Ursae Majoris. 2 $23^h\ 05^m$ 0.31848			γ Ursae Majoris. 2.3 $23^h\ 52^m$ 0.22180			
Latitude.	North altitude	Azimuth.	An. decrease.	South altitude	Azimuth.	An. decrease.	North altitude	Azimuth.	An. decrease.	North altitude	Azimuth.	An. decrease.	Latitude.
°	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	° ′	° ′	″	°
42	4 25	1 29.5	16	17 47	1 02 5	8	14 29	0 59.0	7	6 30	0 39.0	1	42
43	5 24	" 31.0	"	16 47	" 3.5	"	15 29	1 00.0	"	7 29	" 39.5	"	43
44	6 22	" 32.5	17	15 47	" 4.5	"	16 28	" 1.0	"	8 28	" 40.0	"	44
45	7 21	" 34.5	"	14 48	" 6.0	"	17 28	" 2.0	"	9 28	" 41.0	"	45
46	8 20	" 36.0	"	13 48	" 7.0	9	18 28	" 3.0	"	10 27	" 41 5	"	46
47	9 20	" 37.5	18	12 48	" 8.0	"	19 28	" 4.5	"	11 27	" 42.5	"	47
48	10 19	" 39.5	"	11 49	" 9.5	"	20 28	" 5.5	"	12 26	" 43.0	2	48
49	11 19	" 41.5	"	10 49	" 11.0	"	21 27	" 7.0	8	13 26	" 44.0	"	49
50	12 18	" 43.5	19	9 50	" 12.5	"	22 27	" 8.5	"	14 26	" 45.0	"	50
51	13 18	" 46.0	"	8 50	" 14.0	10	23 27	" 10.0	"	15 25	" 46.0	"	51
52	14 18	" 48.5	"	7 51	" 15.5	"	24 27	" 11.5	"	16 25	" 47.0	"	52
53	15 18	" 51.0	20	6 52	" 17.5	"	25 27	" 13.0	"	17 25	" 48.0	"	53
54	16 17	" 53.5	"	5 53	" 19.0	"	26 27	" 14.5	"	18 25	" 49.0	"	54

TABLE VI.

Quantities having relation to the figure and dimensions of the Earth.

Latitude.	One Gunter's chain in seconds of Latitude. (1)	Logarithm. (2)	Difference for 10′ (3)	One Gunter's chain in seconds of longitude. (4)	Logarithm. (5)	Difference for 10′ (6)	Latitude.
° ′	″			″			° ′
42 00	0.6521	9.81429	0.00001	0.8742	9.94161	114	42 00
30	.6520	25		0.8811	.94504	116	30
43 00	.6519	21		0.8882	.94853	118	43 00
30	.6519	18		0.8955	.95208	120	30
44 00	.6518	14		0.9030	.95570	123	44 00
30	.6518	10		0.9107	.95938	125	30
45 00	.6517	06		0.9186	.96312	127	45 00
30	.6517	02		0.9267	.96694	129	30
46 00	.6516	9.81399		0.9350	.97082	131	46 00
30	.6516	95		0.9435	.97476	134	30
47 00	.6515	91		0.9523	.97878	136	47 00
30	.6514	87		0.9613	.98286	139	30
48 00	.6514	84		0.9706	.98702	141	48 00
30	.6513	80		0.9801	.99126	144	30
49 00	.6513	76		0.9899	.99557	146	49 00
30	.6512	72		0.9999	.99995	149	30
50 00	.6512	68		1,0102	1.00441	152	50 00
30	.6511	64		1.0208	.00896	154	30
51 00	.6510	61		1.0318	.01359	157	51 00
30	.6510	57		1.0430	.01830	160	30
52 00	.6509	54		1.0546	.02309	163	52 00
30	.6509	50		1.0665	.02797	166	30
53 00	.6508	46		1.0788	.03294	169	53 00
30	.6507	42		1.0915	.03801	172	30
54 00	.5597	39		1.0045	.04316		54 00

TABLE VI.—*Continued.*

Quantities having relation to the figure and dimensions of the Earth.

Latitude.	One degree of Latitude in chains. (7)	Difference for 10′ (8)	One degree of Longitude in chains. (9)	Difference for 10′ (10)	Logarithm of the convergence of meridians for one chain departure. (11)	Difference for 10′ (12)	Latitude.
° ′	c	c	c	c			° ′
42 00	5520.9	0.2	4118.1	10.8	9.76712	253	42 00
30	5521.4		4085.7	10.9	.77490	253	30
43 00	5521.9		4053.0	11.0	.78255	253	43 00
30	5522.4		4020.0	11.1	.79007	253	30
44 00	5522.9		3986.6	11.2	.79747	252	44 00
30	5523.4		3953.0	11.3	.80503	252	30
45 00	5523.8		3919.0	11.4	.81261	252	45 00
30	5524.3		3884.8	11.5	.82016	253	30
46 00	5524.8		3850.2	11.6	.82775	253	46 00
30	5525.3		3815.4	11.7	.83530	253	30
47 00	5525.8		3780.3	11.8	.84290	253	47 00
30	5526.2		3744.9	11.9	.85045	254	30
48 00	5526.7		3709.2	12.0	.85810	254	48 00
30	5527.2		3673.2	12.1	.86576	254	30
49 00	5527.7		3637.0	12.2	.87335	254	49 00
30	5528.2		3600.4	12.3	.88101	255	30
50 00	5528.6		3563.6	12.4	.88867	256	50 00
30	5529.1		3526.5	12.5	.89637	257	30
51 00	5529.6		3489.1	12.6	.90409	258	51 00
30	5530.1		3451.5	12.6	.91184	259	30
52 00	5530.5		3413.6	12.7	.91962	261	52 00
30	5531.0		3375.5	12.8	.92744	262	30
53 00	5531.5		3337.0	12.9	.93529	263	53 00
30	5532.0		3298.4	13.0	.94318	265	30
54 00	5532.4		3259.4		.95112		54 00